 augmented matrix

3 elementary row operations

Interchange two rows.  This is exactly what it says.  We will interchange row *i* with row *j*.

Multiply row *i* by a constant, *c*.  This means that every entry in row *i* will get multiplied by the constant *c*.

Add a multiply of row *i* to row *j.*  In our heads we will multiply row *i* by an appropriate constant and then add the results to row *j* and put the new row back into row *j* leaving row *i* in the matrix unchanged.

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| A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix},  then the row vectors are **r**1 = (1, 0, 2) and **r**2 = (0, 1, 0). A linear combination of **r**1 and **r**2 is any vector of the form  c_1 (1,0,2) + c_2 (0,1,0) = (c_1,c_2,2c_1).\, | If A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{bmatrix}, then the column vectors are **v**1 = (1, 0, 2)T and **v**2 = (0, 1, 0)T.  A linear combination of **v**1 and **v**2 is any vector of the form  c_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ 2c_1 \end{bmatrix}\, |

Let A be a *m*-by-*n* matrix. Then

1. rank(A) = dim(row(A)) = dim(col(A)),
2. rank(A) = number of pivots in any echelon form of A,
3. rank(A) = the maximum number of linearly independent rows or columns of A.

Let T:Rn→Rm be a linear transformation. The following are equivalent:

|  |  |
| --- | --- |
| **One to one** | **Onto** |
| 1. *T* *is one-to-one*. 2. *T*(**x**)=**0** *has only the trivial solution* **x**=**0**. 3. *If* *A* *is the standard matrix of* *T*, *then the columns of* *A* *are linearly independent.* 4. ker(*A*)={**0**}. 5. nullity(*A*)=0. 6. rank(*A*)=*n*. | 1. *T* *is onto*. 2. *The equation* *T*(**x**)=**b** *has solutions for every* **b**∈R*m*. 3. *If* *A* *is the standard matrix of* *T*, *then the columns of* *A* *span* R*m*. That is: every **b**∈R*m* is a linear combination of the columns of *A*. 4. Im(*A*)=R*m*. 5. rank(*A*)=*m*. 6. nullity(*A*)=*n*−*m*. |
| http://www.regentsprep.org/regents/math/algtrig/ATP5/onetoone.jpg | http://www.regentsprep.org/regents/math/algtrig/ATP5/surjective.gif |

If *A* is an *n* x *n* matrix then http://tutorial.math.lamar.edu/Classes/DE/LA_Eigen_files/eq0015M.gifhttp://tutorial.math.lamar.edu/Classes/DE/LA_Eigen_files/empty.gif is an *nth* degree polynomial.  This polynomial is called the **characteristic polynomial**.

*(W*⊥)⊥ = *W*

(Row *A*)⊥ = Null *A*

(Col *A*)⊥ = Null *A*T

Least square method

Predicted y-value

k0 + k1x1 = y1

k0 + k1x2 = y2

.

.

k0 + k1xn = yn

Xk = y where X = [1 x1 k = [ k0 y = [y1

1 x2  k1] y2

. .

. .

1 xn] yn]

XTXk = XTy

Compute XTX and XTy, then solve for k.

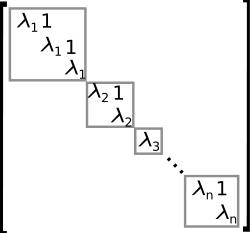
Gram Schmidt

\mathrm{proj}_{\mathbf{u}}\,(\mathbf{v}) = {\langle \mathbf{u}, \mathbf{v}\rangle\over\langle \mathbf{u}, \mathbf{u}\rangle}\mathbf{u} , 


\begin{align}
\mathbf{u}_1 & = \mathbf{v}_1, & \mathbf{e}_1 & = {\mathbf{u}_1 \over \|\mathbf{u}_1\|} \\
\mathbf{u}_2 & = \mathbf{v}_2-\mathrm{proj}_{\mathbf{u}_1}\,(\mathbf{v}_2),
& \mathbf{e}_2 & = {\mathbf{u}_2 \over \|\mathbf{u}_2\|} \\
\mathbf{u}_3 & = \mathbf{v}_3-\mathrm{proj}_{\mathbf{u}_1}\,(\mathbf{v}_3)-\mathrm{proj}_{\mathbf{u}_2}\,(\mathbf{v}_3), & \mathbf{e}_3 & = {\mathbf{u}_3 \over \|\mathbf{u}_3\|} \\
\mathbf{u}_4 & = \mathbf{v}_4-\mathrm{proj}_{\mathbf{u}_1}\,(\mathbf{v}_4)-\mathrm{proj}_{\mathbf{u}_2}\,(\mathbf{v}_4)-\mathrm{proj}_{\mathbf{u}_3}\,(\mathbf{v}_4), & \mathbf{e}_4 & = {\mathbf{u}_4 \over \|\mathbf{u}_4\|} \\
& {}\ \  \vdots & & {}\ \  \vdots \\
\mathbf{u}_k & = \mathbf{v}_k-\sum_{j=1}^{k-1}\mathrm{proj}_{\mathbf{u}_j}\,(\mathbf{v}_k), & \mathbf{e}_k & = {\mathbf{u}_k\over \|\mathbf{u}_k \|}.
\end{align}


|  |  |
| --- | --- |
| Inner product space  \langle x,y\rangle =\overline{\langle y,x\rangle}.  \langle ax,y\rangle= a \langle x,y\rangle.  \langle x+y,z\rangle= \langle x,z\rangle+ \langle y,z\rangle.  \langle x,x\rangle \geq 0with equality only for x = 0. | Cauchy-Schwarz inequality  |\langle x,y\rangle| ^2 \leq \langle x,x\rangle \cdot \langle y,y\rangle,  |\langle x,y\rangle| \leq \|x\| \cdot \|y\|.\, |
|  |  |
|  |  |
| Diagonalizable matrix  A square matrix *A* is called **diagonalizable** if it is similar to a diagonal matrix, i.e., if there exists an invertible matrix *P* such that *P*−1*AP* is a diagonal matrix.  An *n*×*n* matrix A is diagonalizable if and only if the sum of the dimensions of its eigenspaces is equal to *n*  The diagonal entries of this matrix are the eigenvalues of *A*. | Symmetric matrix  A = A^{\top}  For every symmetric real matrix *A* there exists a real orthogonal matrix *Q* such that *D* = *Q*T*AQ* is a diagonal matrix.  Every real symmetric matrix has real eigenvalues. |

Jordan normal form



Any non-diagonal entries that are non-zero must be equal to 1, be immediately above the main diagonal and have identical diagonal entries to the left and below them.

Generalized eigenvector

A generalized eigenvector of *A* is a nonzero vector v, which is associated with λ having algebraic multiplicity *k* ≥1, satisfying (A-\lambda I)^k\mathbf{v} = \mathbf{0}. The set spanned by all generalized eigenvectors for a given λ, form the generalized eigenspace for λ.

Solve for  (A-\lambda I)v_2 = v_1.  where v1 is the first eigenvector, and v2 is the generalized eigenvector, λ is the eigenvalue.

Or solve for (A-\lambda I)^k\mathbf{v} = \mathbf{0}. where **v is the generalized eigenvector**, and k is any integer.

Real roots Complex roots Repeated roots

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*r1* and *r2* http://tutorial.math.lamar.edu/Classes/DE/ComplexRoots_files/eq0003M.gif *r1* = r*2* = *r*

http://tutorial.math.lamar.edu/Classes/DE/RealRoots_files/eq0005M.gif http://tutorial.math.lamar.edu/Classes/DE/ComplexRoots_files/eq0018M.gif http://tutorial.math.lamar.edu/Classes/DE/RepeatedRoots_files/eq0017M.gif

Wronskian and fundamental set of solutions

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Then http://tutorial.math.lamar.edu/Classes/DE/FundamentalSetsofSolutions_files/eq0010M.gif, the 2 solutions are called a **fundamental set of solutions**.

Method of undetermined coefficients

To find a particular solution to the differential equation ay’’ + by’ + cy = Ctmert  use the form

* 1. s = 0 if r is not a root of the associated auxiliary equation
  2. s = 1 if r is a simple root of the associated auxiliary equation
  3. s = 2 if r is a double root of the associated auxiliary equation

To find a particular solution to the differential equation ay’’ + by’ + cy = Ctmeαtcosβt or Ctmeαtsinβt use the form

* 1. s = 0 if α+βi is not a root of the associated auxiliary equation
  2. s = 1 if α+βi is a root of the associated auxiliary equation

|  |  |
| --- | --- |
| *g(t)* | *yp(t)* guess |
| **http://tutorial.math.lamar.edu/Classes/DE/UndeterminedCoefficients_files/eq0030M.gifhttp://tutorial.math.lamar.edu/Classes/DE/UndeterminedCoefficients_files/empty.gif** | http://tutorial.math.lamar.edu/Classes/DE/UndeterminedCoefficients_files/eq0031M.gifhttp://tutorial.math.lamar.edu/Classes/DE/UndeterminedCoefficients_files/empty.gif |
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| *nth* degree polynomial | http://tutorial.math.lamar.edu/Classes/DE/UndeterminedCoefficients_files/eq0038M.gif |

Variation of parameters

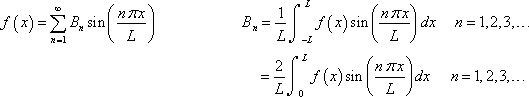
ay’’+by’+c = g(t)

yp(t) = v1(t)y1(t) + v2(t)y2(t)

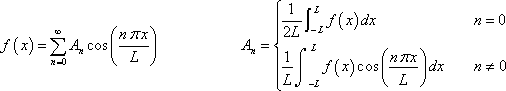
y1v1’ + y2v2’ = 0

y1’v1’ + y2’v2’ = g/a

Fourier sine series



Fourier cosine series



Heat equation

\displaystyle u_t = \alpha u_{xx}

u(x,0) = f(x) \quad \forall x \in [0,L] u(0,t) = 0 = u(L,t) \quad \forall t > 0 

\displaystyle u(x,t) = X(x) T(t).

\frac{\dot{T}(t)}{\alpha T(t)} = \frac{X''(x)}{X(x)}. both sides are equal to some constant value −λ

\dot{T}(t) = - \lambda \alpha T(t)

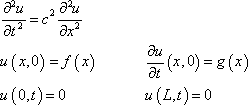
X''(x) = - \lambda X(x).

|  |  |  |
| --- | --- | --- |
| λ < 0 | λ = 0 | λ > 0 |
| X(x) = B e^{\sqrt{-\lambda} \, x} + C e^{-\sqrt{-\lambda} \, x}. | *X*(*x*) = *Bx* + *C* | X(x) = B \sin(\sqrt{\lambda} \, x) + C \cos(\sqrt{\lambda} \, x). |

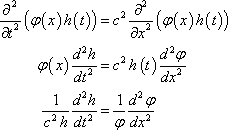
T(t) = A e^{-\lambda \alpha t} \sqrt{\lambda} = n \frac{\pi}{L}.

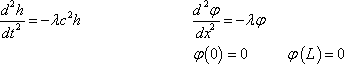
u(x,t) = \sum_{n = 1}^{\infty} D_n \sin \left(\frac{n\pi x}{L}\right) e^{-\frac{n^2 \pi^2 \alpha t}{L^2}} D_n = \frac{2}{L} \int_0^L f(x) \sin \left(\frac{n\pi x}{L}\right ) \, dx.

Wave equation



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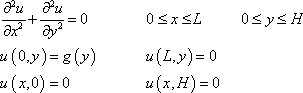


Solution:

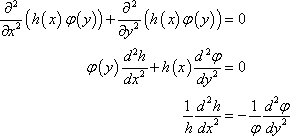
Where an’s and bn’s are determined from the Fourier sine series

d’Alambert solution

Laplace’s equation

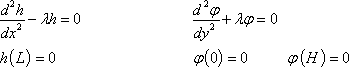


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