Astro160
Fall 2015
MT Review

$\text{Flux} = \sigma T_{eff}^4$ (Stefan-Boltzmann)

* Snapshots → infer evolution
* Mass, composition determine entire lifecycle of star
* star spherically symmetric, isolated effects, uniform initial composition...

$r \propto \frac{1}{n^{1/3}}$

$P_{gas} = nkT$ (classical) ✓ ideal gas, (no particle mass) "noninteracting", nonrelativistic, elastic collisions

$P_{QM} = \frac{h^2}{5 m_e} n_e^{5/3} \left(\frac{3}{8\pi}\right)^{2/3} = 5\times10^{16}\ \text{erg/cm}^3$ (in sun)
(non-relativistic)

$\frac{dM_r}{dr} = 4\pi r^2 \rho$

Sun ☉
$L_\odot = 4\times10^{33}$ erg/s
$R_\odot = 7\times10^{10}$ cm $= 10 R_J$
$M_\odot = 2\times10^{33}$ g $= 10^3 M_J = 10^5 M_E$
$T_{eff\odot} = 5800$K
$T_{c\odot} = 1.5\times10^7$K
$\langle\rho\rangle_\odot = 1$ g/cm³ $= \rho_{H_2O}$
$\rho_{c\odot} = 150$ g/cm³

$a_0 \sim 5\times10^{-9}$ cm bohr radius

Force Balence ($P = P_{gas} + P_{rad} + P_{deg}$)

Look at Hydrostatic Equilibrium

$\rho a = -\frac{dP}{dr} - \rho \frac{GM_r}{r^2}$

at $a=0$ (HE), $\frac{dP}{dr} = -\rho\frac{GM_r}{r^2}$

$t_{dyn} \sim \frac{1}{\sqrt{G\langle\rho\rangle}}$ ("freefall") 1hr

$t_{KH} \sim \frac{U^{\text{grav energy}}}{L}$ (radiate all energy) 30 million yrs

$t_{age} \sim \frac{E_{fusion}^{\text{binding energy}}}{L}$ (burn all Hydrogen) 5 billion yrs

Isothermal Atmosphere ($P = nkT$)

$h \sim \frac{P}{\rho g} = \boxed{\frac{kT}{mg}}$

Scaleheight: $\Delta P = P$, $\Delta r = h$

$P_{neutral\,H} = nkT$

$P_{ionized\,H} = 2nkT$ (H protons + H electrons)

$\mu = \frac{m_{mean}}{m_{H\,atom}}$

$\frac{1}{\mu_e} = \sum_i \frac{Z_i X_i}{A_i}$ (X_i mass fraction, A_i weight)

$P = \frac{\rho kT}{m_p \mu}$

$\frac{1}{\mu_{ion}} = \sum_i \frac{X_i}{A_i}$

$\left(n_e = \frac{\rho_e}{m_p\mu_e}\right)$

Virial thm

$-E = K = -\frac{1}{2}U$ (star single, bound stars)

$U = 3\langle P\rangle V$ (NR)

$U = -K$ $\boxed{E=0}$ (relativistic thm) $M > 100 M_\odot$, photons...

photons "blackbody" (obeys planck fn)

$U = aT^4$ $\sigma = \frac{ca}{4}$ (like SB law!)

$P_{photons} = \frac{1}{3}aT^4$

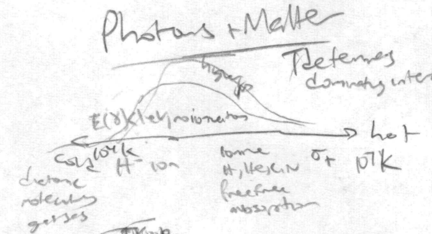
KH contraction

$E_{tot} = -K$, so $K\uparrow \Rightarrow |U|\uparrow \Rightarrow T\uparrow \Rightarrow R\downarrow$

don't burn out, since fusion

Photons + Matter

T determines dominating interaction!



Star formation

$\langle n\rangle \approx 1$ particle/cm³
$\langle\rho\rangle \approx 10^{-24}$ g/cm³

Hot ISM $n \sim 10^{-2}$ cm⁻³ 10^6K
Ionized gas $n \sim 1$ cm⁻³ 10^4K } most volume
neutral atomic H ~ 100 cm⁻³ 100K
H_2 molecular gas $\sim 10^4$ cm⁻³ 10K } most mass

Point: molecular

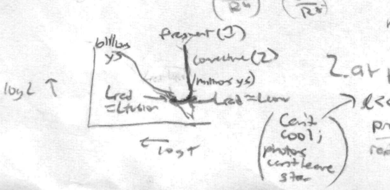
gravitational instability $M_J \simeq 50 M_\odot \frac{(T/10K)^{3/2}}{(n/1000\text{cm}^{-3})^{1/2}} \propto \frac{1}{\rho^2}$ denser ⇒ more likely to collapse

then, $t_{ff} \simeq \frac{1}{\sqrt{G\rho}} \sim 10^7 \text{yrs} \left(n/100\text{cm}^{-3}\right)^{-1/2}$

Force Balence $\rho \vec{a} = \underbrace{\frac{nkT}{R}}_{gas} - \underbrace{\rho\frac{GM}{R^2}}_{gravity}$ $\left(\frac{MT}{R^4}\right)$ $\left(\frac{GM^2}{R^5}\right)$

$T\uparrow\downarrow \Rightarrow$ balence

1. first, ρ low, ℓ large → cools efficiently, $T \simeq$ const
gravity wins
⇒ runaway contraction
$n\uparrow \Rightarrow M_J\downarrow \Rightarrow$ fragmenting!
$n\uparrow \Rightarrow t_{ff}\downarrow$ → fragments collapse faster!!



(Can't cool; photons can't leave star)

2. at HE, stops contracting
→ $\ell \ll R$, adiabatic, S = const on Hayashi Line $\boxed{\tau = 10^6 \text{yrs} \left(\frac{M}{M_\odot}\right)^{-2}}$
pressure wins, $M_J\uparrow$ (stops) (stops when $L_{fusion} = L_{rad}$)
radiating energy as blackbody!
⇒ KH contraction! model as polytrope
$L_{conv} > L_{rad}$, $\boxed{\text{convective}}$ $(R \propto t^{-1/3})(L \propto t^{-2/3})$

Prad vs Pgas

$P_{rad} = \frac{1}{3}aT^4$
$P_{gas} = nkT$, $kT = \frac{GM\mu m_p}{3R}$ (by virial thm) (btw) ($T \propto M/R$)
$\Rightarrow L \propto M^3$, $\propto T^6$, $R \propto M^{0.75}$

so, $P_{rad} > P_{gas}$ at $M_{critical} \approx 100 M_\odot$

not bound! since $E \approx 0$ (relativistic virial thm)

from flux eqn, $L_r \ll L_{edd}$, $L_{edd} = \frac{4\pi GM_r c}{\kappa_m}$

$\boxed{\text{Energy Transport}}$ $\left(\text{then } \frac{dP_{rad}}{dr} \ll \frac{dP_{gas}}{dr}\right)$ $L_{edd} \propto M$, $L \propto M^3 \Rightarrow M \approx 100 M_\odot$, $L = L_{edd}$

* $\vec{F} = -\chi\nabla T$ $\left(\chi = \frac{1}{3}v\ell\frac{dU}{dT}\right)$

Radiation

$\ell = \frac{1}{n\sigma} = \frac{1}{\kappa\rho}$

electrons conduction: $v_{th} \sim \sqrt{kT/m}$ significant deflection at $\sigma = \pi b^2$, $b \sim e^2/kT$
$\sigma = \frac{10\pi e^4}{(kT)^2}$ (high T) high T, but negligible for sun
($P_{rad} \ll P_{gas}$ but $F_{rad} \gg F_{cond}$) $\kappa \propto T^{5/2}$, $\frac{1}{\sqrt{m}}$ coulomb scattering ($\ell_e \sim \ell_p$ but $v_e \gg v_p$ so $\chi_e \gg \chi_p$)

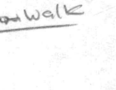
photons radiation: $v = c$, $U = aT^4$, $F = -\frac{4}{3}\ell\nabla(\sigma T^4)$ ($\ell_{photon} = 10^6 \ell_e$)
Thomson scattering (high T) $\sigma_T \propto \frac{1}{m^2}$ $\boxed{\kappa = 0.4\text{cm}^2/\text{g}}$

$\boxed{\text{This is it}}$ Right Ballpark $\boxed{\text{photon random walk sets } L \text{ in star}}$

$t \sim$ typical time $\sim \boxed{R^2/\ell c}$ Random walk

$t_{KH} - \frac{U_{th}}{U_{rad}}t = \frac{nkT}{aT^4}\frac{R^2}{\ell c}$

Fully Convective Stars



Since $P \propto \rho^\gamma$ for QM degeneracy pressure

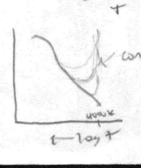
Polytropic models $P = K\rho^\gamma$
differential eq for star
$n = \frac{1}{\gamma-1}$ Φ, ξ, a,
$(K, \rho_c) \to \rho, P$

$\boxed{\text{But can't infer properties of sun! Just a MODEL}}$

(so, Mass only parameter)

$L \propto M^{4/7}R^2$ $0.2L_\odot$ $T_{eff} \approx 4000$K Hayashi Line

Higher mass → eventually radiative core so less time convective



Convection

$T ds = dU - \frac{P}{\rho^2}d\rho$

= 0 when adiabatic: $P = \Phi U \rho$ by virial thm, $\Phi = 2/3$ NR, $1/2$ rel

$U \propto \rho^\Phi$, $P \propto \rho^{\Phi+1}$, $\gamma = \Phi + 1$

NR, $T \propto \rho^{2/3}$ $\boxed{S\ \text{const} \Rightarrow P \propto \rho^\gamma}$

Instability → $t_{convection} \ll t_{rad} = t_{KH}$

$s + s_{blob} \approx s$

$\frac{ds}{dr} < 0$ → pressure balance
(Schwarzschild criterion) $t_{dyn} \ll t_{convection}$
$P_{blob} = P_{amb}$

$t \sim 1/N$ brunt vaisala freq ~20 days

$\frac{d\ln T}{dr} > \frac{\gamma-1}{\gamma}\frac{d\ln P}{dr}$

$\frac{1}{4}\frac{P}{P_{rad}}\frac{L_r}{L_{edd}} > \frac{\gamma-1}{\gamma}$

$\kappa\uparrow$, $T\downarrow \Rightarrow$ more convection $M \simeq 1/3 M_\odot$ (L_{edd})

Mixing length: $\ell = \alpha H$

$\boxed{F_c} = \rho v_c^3 = \rho\alpha^3 c_s^3 \left|\frac{H}{c_p}\frac{ds}{dr}\right|^{3/2}$

convection ≈ adiabatic $P \propto \rho^\gamma$, $T \propto \rho^{\gamma-1}$

so, $\frac{1}{T}\frac{dT}{dr} = \gamma - 1\,\frac{1}{\rho}\frac{d\rho}{dr}$ $(F_c \ll \rho c_s^3)$

Fusion valley of stability $Z=N$ 1:1 $N>Z$ heavy

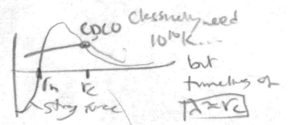
$Z, N, A = Z+N$

nuclear force at short distances $\lesssim r_n \sim 10^{-13}$ cm

$\rho_n \sim 2\times10^{14}$ g/cm³

QFT → QED, QCD
massless photons, strong nuclear short range, massive pions

$t_{age} \sim \frac{E_{fusion}}{L_\odot} \sim 10^{11}$ ys



$E = \frac{1}{2}m_r v^2 + \frac{Z_1 Z_2 e^2}{r}$

KE, potential

$\left(T \sim (10^7 K) Z_1^2 Z_2^2 \right.$ to have tunnelling, need to be hotter to have higher elements fused!!

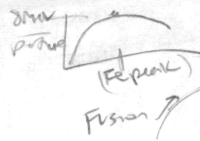
Binding Energy

$n \to p + e^- + \bar{\nu}_e$

binding energy E_b

$M_{nucleus} = Z m_p + N m_n - E_b/c^2$

7 MeV (He) per weight



min mass energy at Fe, Ni

Fusion, Fission

Fermi energy $E_F \approx 25$ MeV
$E \propto \frac{1}{r} \approx 10^5$ E bytes... } MeV

either need high T or high ρ (cold fusion)

tunnelling

Coulomb's potential

QM problem

spherical

$Y_{lm}(\theta,\phi) F_l(r)$

dominated by $l=0$

since barrier so large!! $f'' + g f = 0$

use WKB approx (roughly constant potential over wavelength)

$\psi(r) = e^{\pm i \int \sqrt{g}\, dr}$ $\boxed{(\theta')^2 = g(r)}$

need $\frac{1}{\sqrt{g}} \ll g/g'$

$\lambda \ll L$ $\lambda = h/p$
wavelength, potential changes

$\boxed{E_g = 1\,\text{MeV} \frac{m_r}{m_p} Z_1^2 Z_2^2}$

high tunneling $E \geq E_g$, $E \leq E_g$ fusion favors high energy particles

$\boxed{P \simeq e^{-\pi r_c/\lambda} = e^{-(E_g/E)^{1/2}}}$

P(tunneling) depends on r_c

But (Not on r_n, products of rxns)

$\langle E \rangle = 2$ keV, $\langle E_G \rangle = 500$ keV
(but look at boltzmann dist) $P(E_G) = 10^{-7}$ $P(10 \langle E \rangle) = 10^{?}$

$\boxed{\sigma = \text{tunnel chance} \times \text{binding chance}}$

$\sigma \sim \pi\lambda^2 \frac{S(E)}{E}$ ~ strong force, weak force, other physics

$\sigma_T = 0.6$ barn

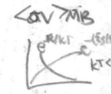
$S_{strong} \sim 1$ keV barn (10^{-24} cm²)

$S_{weak} = 10^{-20}$ keV Bm

$\Gamma_{12} = n_1 n_2 \sigma v$

reactions / time, volume

$\langle \sigma v \rangle_{MB}$



$\langle \sigma v \rangle = 2.6\, S(E_0) \frac{E_G^{1/6}}{(kT)^{2/3}\sqrt{m_r}} e^{-3(E_G/4kT)^{1/3}}$

$\epsilon = \frac{r_{12}}{\rho} Q \ldots = \frac{n_1 n_2 \langle \sigma v \rangle}{\rho} Q$

$n_i = \frac{\rho}{m_i} x_i$

$\epsilon_{12} \propto \frac{\rho}{T^{2/3}} e^{-3(E_G/4kT)^{1/3}}$

(eqn) $\epsilon \propto \rho T^{\beta}$ $\beta = -\frac{2}{3} + \left(\frac{E_G}{4kT}\right)^{1/3}$

(1) CNO - strong but E_G high (45.7 MeV)

(2) PP - weak force E_G (0.5 MeV)

both appropriate at $M \approx M_\odot$

He Fusion (PP)

PP $p + p \to {}^2H + e^+ + \nu_e$ (weak!!)

$L \sim M\epsilon$ $\epsilon \propto \rho T^4$

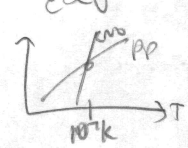
$\approx \frac{10^7 M_\odot}{(T/10^7K)^{2/3}} e^{-15.7 (T/10^7K)^{-1/3}} \epsilon$

CNO $\boxed{L_{CNO} = 0.65 L_\odot}$

$\epsilon \propto \rho T^{20}$

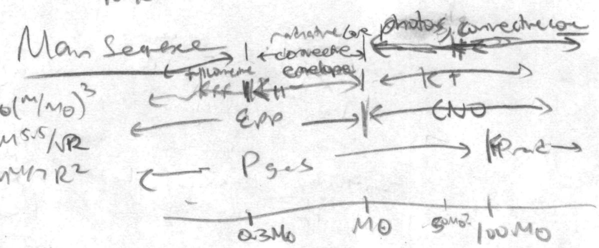
based on ~~abundances~~

heaviest mono: Sun, ...



10^7 K

Main Sequence



$K_T = $ const $L \sim L_\odot (M/M_\odot)^3$

$K_{ff} \propto \rho T^{-7/2}$ $L \propto M^{5.5}/\sqrt{R}$

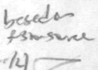
convective $L \propto M^{4/7} R^2$

Regions 0.3–1 $M_\odot$ $R \propto M^{1/7}$, $T_c \propto M^{4/7}$, $L \propto M^{5.5}$ $L \propto T_{eff}^4$

1–50 $M_\odot$ $T_c \propto M^{0.2}$, $L \propto M^3$ $T_{eff} \propto M^{3/4}, L^{1/4}$
$R \propto M^{0.8}$

50+ $M_\odot$

convection when $\frac{M_r}{M} < \frac{5}{8} \frac{P}{P_{rad}} \frac{L}{L_{edd}} \frac{L_r}{L}$ — power in fusion, changes based on fusion source



Sun: $\frac{M_r}{M} \simeq 0.1$ then

but $M > 1.2 M_\odot$ ⇒ CNO fusion, convective core!

Lifetime on main sequence

$t_{ms} \simeq 10^{10}$ ys $(M/M_\odot)^{-2.5}$

Min masses

look at QM degeneracy pressure

$\lambda \sim \frac{h}{p} \sim \frac{h}{\sqrt{mkT}} > n^{-1/3}$

$n \geq n_Q$, $n_Q = \left(\frac{2\pi m kT}{h^2}\right)^{3/2}$

then QM matters

$M < M_\odot \Rightarrow T\downarrow, n\uparrow \Rightarrow n_Q\downarrow, n\uparrow$ QM more important!!

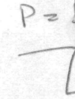
degenerate $n(p) \propto e^{-E_p/kT}$

$E_F = \mu = \frac{h^2}{2m}(3\pi^2 n)^{2/3}$ $\boxed{P = npv}$

$E_F = p_F p_{max}$

$P = \frac{hc}{4}\left(\frac{3}{8\pi}\right)^{1/3} n^{4/3}$ ultra relativistic

$P = \frac{h^2}{8m}\left(\frac{3}{8\pi}\right)^{2/3} n^{5/3}$ NR

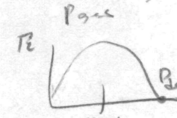


$\left(n = \frac{8\pi}{3h^3} p_{max}^3\right)$

Fermi-Dirac $n(p) = \frac{2/h^3}{e^{(E_p-\mu)/kT}+1}$

high mass stars → less dense in cores ⇒ more radiation per mass

$P = nkT + \frac{h^2}{5m_e}\left(\frac{3}{8\pi}\right)^{2/3}\left(\frac{\rho}{\mu_e m_p}\right)^{5/3}$
P_{gas}, P_{deg}

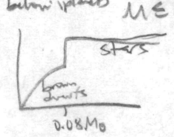


no self gravitating object $> 8\times10^{6}$ K $(M/M_\odot)^{4/3}$

KH contraction
$R\downarrow \to T\uparrow, \rho\uparrow$ either fusion (T_c constant)
or $\rho \to P_{rem}$ (balance gravity) no more contraction brown dwarfs / giant planets

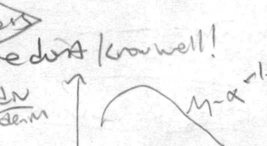
$\boxed{T_c \sim 2\times10^6 K}$ above is star, below is planet

$M \geq 0.06 M_\odot$ star
$M \leq 0.06 M_\odot$ brown dwarf



Med Mass

we don't know well!



(Most stars low mass)
(most luminous high mass)

1. unstable (virial thm) ($E_{tot} = 0$)

2. can't form, L too high, blows away matter ($L \propto L_{edd}$)