**Navier-Stokes**

**Equation of continuity**

|  |  |
| --- | --- |
| \begin{align}   r:\ &\rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} +                    \frac{u_{\phi}}{r} \frac{\partial u_r}{\partial \phi} + u_z \frac{\partial u_r}{\partial z} - \frac{u_{\phi}^2}{r}\right) = {}\\       &-\frac{\partial p}{\partial r} + \mu \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u_r}{\partial r}\right) +         \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \phi^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} -         \frac{2}{r^2}\frac{\partial u_\phi}{\partial \phi} \right] + \rho g_r \\   \phi:\ &\rho \left(\frac{\partial u_{\phi}}{\partial t} + u_r \frac{\partial u_{\phi}}{\partial r} +                       \frac{u_{\phi}}{r} \frac{\partial u_{\phi}}{\partial \phi} + u_z \frac{\partial u_{\phi}}{\partial z} + \frac{u_r u_{\phi}}{r}\right) = {}\\          &-\frac{1}{r}\frac{\partial p}{\partial \phi} + \mu \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u_{\phi}}{\partial r}\right) +            \frac{1}{r^2}\frac{\partial^2 u_{\phi}}{\partial \phi^2} + \frac{\partial^2 u_{\phi}}{\partial z^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \phi} -            \frac{u_{\phi}}{r^2}\right] + \rho g_{\phi} \\   z:\ &\rho \left(\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_{\phi}}{r} \frac{\partial u_z}{\partial \phi} +                u_z \frac{\partial u_z}{\partial z}\right) = {}\\       &-\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u_z}{\partial r}\right) +         \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \phi^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z. \end{align} | **Equation of continuity**   \frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(\rho r u_r\right) +   \frac{1}{r}\frac{\partial (\rho u_\phi)}{\partial \phi} + \frac{\partial (\rho u_z)}{\partial z}     = 0.  |
| **Assumptions** | **Implications** |
| Flow is steady, d/dt = 0 |  |
| Plates are infinite in 1. x/y direction
2. x/z direction
3. y/z direction

Parallel flow |  |
| Incompressible, Newtonian, laminar, constant properties |  |
| No pressure gradient |  |
| 2D | 1. , ,
2. , ,
3. , ,
 |
| Gravity acts in the –x,-y, or -z direction |  |
| No slip at wall |  |
| At the free surface (*x =* *h*), there is negligible shear |  |
| The pipe is infinitely long in the z-direction, parallel flow |  |
| The velocity field is axisymmetric with no swirl  | *u*θ = 0 and all partial derivatives with respect to θ are zero |

**Conduction**

k – thermal conductivity, in units of power per distance per temperature (Btu / (hr ft F)

α – thermal diffusivity, in units of area per time (ft^2 / hour)

Thermal Resistance

Plane: Cylinder: Sphere:

Series resistance: Parallel resistance:

**Non-steady state conduction**



Ts – constant average temperature of surface

Tb – average temperature of the slab/sphere at time t or at position r

Ta – initial temperature of the slab/sphere

**Figure 10.5** Average temperatures during unsteady-state heating or cooling of a large slab, and infinitely long cylinder, or a sphere.

|  |  |
| --- | --- |
|  | For infinitely long (no end effects) cylinder:For a sphere: |

Biot number:

**For constant surface**

**To get time, based on all temperatures**

Get θ 🡪 Fig 10.5 to Get F0 🡪 Get time

**To get average temperature, based on time**

Based on time, get F0 🡪 Fig 10.5 t go get θ

**For convective film**

**To get time, based on all temperatures**

Calculate Re and Pr 🡪 Get Nu 🡪 Get h 🡪 Get Bi && Get θ 🡪 Fig 10.7/10.8 to Get F0 🡪 Get time

**To get average temperature, based on time**

Calculate Re and Pr 🡪 Get Nu 🡪 Get h 🡪 Get Bi && Get F0 🡪 Fig 10.7/10.8 to get θ based on F0 and Bi



**Figure 10.7** Change with time of the average temperature of a slab with external convective resistance.



**Convection**

h is the individual heat transfer coefficient, obtained from empirical methods, power per area per temperature

For flow in pipes, 0.6<NPr<16700

|  |  |
| --- | --- |
|  | External forced convection normal to tubes |
|  | Flow past single sphere |

Where kw is the thermal conductivity of the pipe wall and is the wall thickness xw

**Concentric pipe**



If





h- shell

c- tube

a- in

b- out







