CE C30 Final Exam Practice Problem Procedures

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December 10, 2014

1 Problem 1

1.1 Determining Strain Energy

After finding the lengths of each rod using trigonometry, we use the **method of joints** to solve for each of the reaction forces. We then use these forces, along with the corresponding lengths and areas, to calculate the sum of the system's *internal* energy.

1. Use the method of joints, first using the ΣF for x and y at the joint where the load is being applied, to solve for reaction forces F_{CD} and F_{BC} (the two 'standing edges' of the triangle) in terms of the load, P.
2. Solve at joint D to find the force of the 'bottom edge', F_{BD} of the triangle in terms of the load, P.
3. Plug these loads (F_{CD}, F_{BC}, and F_{BD}, along with the corresponding areas and lengths of each of the members, into the formula $U_i = \frac{F^2L}{2EA}$.
4. Sum up these values and that's your answer!

1.2 Find the vertical displacement at point C

We know that the total internal energy we found has to be equal to the external energy, which equals $\frac{1}{2}F\delta_E$) Therefore, because we found the external energy in terms of P (after we solved for all the member forces in terms of P), and the external load being applied at point C is P (i.e. F = P and $\delta_C = \delta E$, we can write $\frac{1}{2}P\delta_E = \frac{0.233P^2L}{EA}$ and solve for δ_E. This is our deflection.

1.3 Find the horizontal displacement at point D

Here, we plug into the axial loading deformation equation $\delta = \frac{FL}{AE}$, where F is F_{BD}, which we solved for in terms of P earlier in the equation, L is the length of rod BD, A is the area, and E is the Elastic Modulus.

1.4 What is P_{cr}?

To find the critical load, realize that it is going to be either the load on BC or CD that will buckle, so substitute those lengths, along with the given value, into the equation that the homie Shaofan gives us.

2 Problem 3

2.1 Draw Mohr's circle of the stress state at that point

Get out the paintbrush – it's time to become an artist and draw a cool circle. And people thought engineers were square. Anyways, draw a circle. If you can't do that, refer to that one Spongebob episode where Squidward thinks he knows art.

The below figure is going to be helpful, Note that most of the below formulas are *given*, so it's really a matter of knowing which of the 4 values in the stress matrix correspond to which variables and knowing how to draw the circle once these values have been calculated. If you were to get a tattoo between now and Wednesday, I'd recommend getting this:

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$$

Steps to Draw:

1. First, offset the center of your circle by σ_{ave}, which is $\frac{\sigma_x+\sigma_y}{2}$. In this case, it was zero, so the circle's center remained at the origin.
2. Find the $tan(2\theta_p)$ value with $\frac{\tau_{xy}}{\frac{1}{2}(\sigma_x-\sigma_y)}$.

   Remember to take the tan^{-1} to find the actual angle!
3. Use this angle and draw a line R, whose length is $\sqrt{(\frac{\sigma_x-\sigma_y}{2})^2+(\tau_{xy})^2}$ with respect to this angle to the horizontal.
4. Label points X and Y with $(\sigma_x, -\tau_{xy})$ and (σ_y, τ_{xy}), respectively.

2.2 Find principal stresses σ_1 and σ_2 and show the results on properly oriented element in physical space

The principal stresses can be found by the equation:

$\sigma_{max,min} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{(\frac{\sigma_x-\sigma_y}{2})^2+(\tau_{xy})^2}$, or, in English, the average stress plus R for the maximum stress, and the average stress minus R for the minumum. In this case, the average stress was zero, so we just got plus R for the maximum and minus R for the minimum

To draw the square (maybe engineers are square):

1. First draw a horizontal line.
2. Then, draw the top of your square rotated θ_p) degrees with respect to that line (**note**: this is **not** the absolute value, so if your angle is negative, it has to be below the horizontal!).

3. Now, draw the minimum principal stress applied to the left and right sides of the square. **If it is a negative value, it must be pointing inward!**

4. Draw the maximum principal stress on the top and bottom.

2.3 Find the maximum shear stress, and show the results on a properly oriented element in physical space

This is similar to before, except now you have to find a new angle, θ_s.

1. Find $2\theta_s$ with 90 - | $2\theta_p$ | (note that it is the **absolute value** of $2\theta_p$).

2. Because our $2\theta_p$ was negative, make $2\theta_s$ negative.

3. Divide $2\theta_s$ by 2 to get θ_s.

4. Draw a vertical line.

5. Draw a square rotated θ_s from the vertical. This will be to the left if it is negative, similarly to how it was below the line if it was negative for the previous part.

6. The maximum shearing stress, τ_{max} is simply your R from before.

7. Your σ' is equal to $\sigma_{ave} = \frac{\sigma_x+\sigma_y}{2}$

8. On the each side of the square, draw an arrow parallel to the side, labeled with shearing stress, and one perpendicular to the side, indicating σ'.

3 Problem 4

Solving this type of problem all boils down to one equation: $EIy'' = M(x)$. In this case, y is equal to the deflection, its first derivative y' is equal to the rotation at the given cross-section, and y" is equal to the curvature at the given cross-section ($1/\rho$). Therefore, to find the deflection y itself, we have to integrate twice, using boundary conditions to solve for the constants.

Before we can do any of this, however, we have to determine what M(x) is in the first place. Here are the steps:

1. As stated in the hint, make a cut at a point 'x'.

2. Take the moment about the cut and solve for M(x):

   ΣM_{cut} = -M(x) -Py(x) = 0 $\rightarrow$ M(x) = -Py(x)

   Note: in the above equation, y is the distance that the beam has been displaced from its original axis. Obviously, as we move down the beam, this amount will vary, which is why it is a function of x.

3. Since $EIy'' = M(x)$, we know that $EIy'' = -Px$. Move both sides to the left of the equals sign to get y" + $\frac{Py}{EI} = 0$

4. Notice how y is the only thing that varies in the second term of the above equation. Therefore, let us set a constant, δ equal to $\sqrt{\frac{P}{EI}}$ so that δ^2 is equal to $\frac{P}{EI}$

5. Plug this into the solution: y = Asin(δx) + Bcos(δx).

6. Apply the boundary conditions.

   (a) Since y(0) = 0, and cos is 1 at 0, B must be 0.

   (b) Since y'(L) = 0, δAcos(δL) = 0. This means that either A = 0, which means that we have a trivial solution, or it means that cos(γL) = 0.

   (c) We know that cosine is equal to zero on multiples of $\frac{pi}{2}$, so we set δL = $\frac{n\pi}{2}$.

7. We then divide both sides by L and square δ to get $\delta^2 = (\frac{n\pi}{2L})^2$.

8. We substitute back in for δ the value of $\frac{P}{EI}$ and solve for P, plugging the lowest possible value of n – 1 – to get P_{cr} in terms of E, I, and L.

   Recall that L_e = 2L in this case!

9. Calculate I_z and I_y, using the parallel axis theorem on the former.

10. Plug both I_z and I_y into the formula for P_{cr}. The one that is lower is your answer!

4 Problem 6

4.1 a. Draw shear diagram

Come on now, you got this.

4.2 b. Find maximum nail spacing Δ_s

Once you've solved for q in the first hint given (this requires calculation of Q and I), set it equal to $N_{Allowable}$ and solve for Δ_s.

1. Find the Q of the tiny 4 by 2 mm bar in the cross-section shown. A is simply 8 mm (4 times 2), and y is simply the distance of that tiny bar's centroidal axis from the centroidal axis of the entire beam, which is 3mm.

2. Find I_z by subtracting the moment of inertia of the small (nonexistent) square from that of the larger square in the cross-section.

3. Now, with the V being the 1000 N/m distributed load, we can calculate q with $\frac{VQ}{I_Z}$.

4. Manipulate the third equation, turning it into q$\Delta_s = N_{allowable}$.

5. $N_{allowable}$ is actually $2F_{nail}$, so plug in the given value for F_{nail} of 200 N and solve for s, the spacing!

5 Problem 7

6 a. Find the shear stress distribution along the section of the shaft

Obtain the shear stress as a function of r with the equation $\tau(r) = \frac{G\phi}{L}r$ for each material.

7 b. Find the angle of twist of the shaft

1. Realize that since the torque on each material i is $\frac{G_i J_i}{L_i}\phi_i$, the total torque must be the torque applied T and is the sum of the two torques: T $= (\frac{G_{aluminum}J_{aluminum}}{L} + \frac{G_{steel}J_{steel}}{L})\phi$.

2. Since we know all of these values except for ϕ, we rearrange this equation to $\phi = \frac{TL}{G_{aluminum}J_{aluminum}+G_{Steel}J_{Steel}}$.

3. Find J for both aluminum and steel with the equation $J = \frac{\pi r^4}{2}$, making sure to use r_{inner} for the inner circle and subtracting the J calculated with r_{inner} from the J calculated with r_{outer} for the outer material.