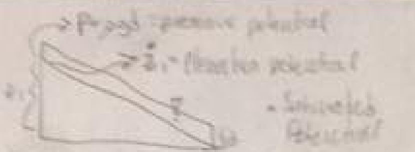


Here $q = -K_{sat}\frac{dh}{dx}$, over a 100m long section, the surface of the water table has $h = \sqrt{100 - x}$ → $K_{sat} = 1\%$
a) where does Darcy Flux vary w/ x?

$\rightarrow q = \frac{0.1}{2\sqrt{100-x}}$

b) Is depth of aquifer at x @ steady state?

$\rightarrow$ if ss → $\frac{\partial h}{\partial t} + \frac{\partial(q_x h)}{\partial x} = 0$

$q_x(h) = \frac{0.1\sqrt{100-x}}{2\sqrt{100-x}} = 0.05 \rightarrow \frac{\partial}{\partial x}(0.05) = 0$ ✓

Mass balance in unconfined aquifer:

$\frac{\partial(\phi_e \delta x\, h)}{\partial t} = (q_x(x)\, h(x)) - q_x(x+\delta x)\, h(x+\delta x))$

$\phi_e \frac{\partial h}{\partial t} = K_{sat}\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) + i$

Steady State: $K_{sat}\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) = -i$

$\frac{\partial(h)}{\partial x} = \frac{1}{2}\frac{\partial}{\partial x}(h^2)$, $\frac{K_{sat}}{2}\frac{\partial^2}{\partial x^2}(h^2) = -i$

$\rightarrow h^2 = -\frac{ix^2}{K_{sat}} + C_1 x + C_2$

apply $h(0) = h_0$, $\frac{\partial h}{\partial x}(L/2) = 0$.

Flow 4 unsaturated:
what rate of vertical flow (f_c) = infiltration capacity

$\Delta P = \frac{2\sigma}{r}$ → surface tension of H_2O → radius of pore

Mat pot = 0 in all sat soils

Now $q = -K\theta\nabla h$ (Ψ = potential in unsat)

unsat	sat	
pressure head = 0	pressure pot > P_atm	
matric < 0	mat pot = 0	

mat potential = same over a vertical distance of 10cm $= \frac{150m}{.1m} = 1500m$

$= \frac{A}{2}t^{1/2} + K_{sat}$ → sorptivity; till t_p, $f_c > p$ → all rain infiltrates, after t_p some does not inf

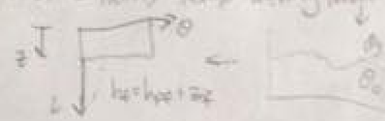
Definitions:
aquifer = groundwater
watershed = area of land draining to a single point
hydrograph = ...
Unconfined aquifer = open to surface (top)
confined A = below impermeable layer (above)
Increase in deep aquifer cause rise in wt. separate rainfall change to match rate of rise

ϕ_e = effective porosity ($\leq \phi$ b/c pores are not empty)

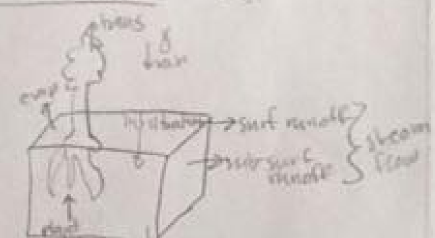
salinity effects density!! $q = -K\theta\nabla h$
unsaturated soil → matric potential less than ref potential
drier soil → more negative matric pot.

Infiltration → rapid in wet, ~ slow in unsat

G-A Assumptions → soil is uniformly wet & there's sharp wetting front



"water content" = $\sim\theta_s$



– in soil: $\phi_E\frac{\partial s}{\partial t} = f - D - ET - Q$ → lat fluxes
infil → deep drainage, evapotrans

$f\frac{\partial h}{\partial t} = K_{sat}\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) + i$ → sm k/ounce
porosity, depth of water table

$\frac{\partial s}{\partial t} = -\frac{\partial}{\partial x}(Aq) + i$ → av flow rate, → source/sink
x-sectional area carrying flow

assume flat aquifer → ϕ explains # water vs. throu

– water in rivers/streams:
quick/storm-flow = immediate response to rainfall
baseflow = normal

$h = z' + \frac{P_w}{\rho_w g}$

to reach t_p, $P > K_s$

$F > z_f(\theta_s - \theta_o)$ → depth of infiltrated H_2O

$q = f = \frac{dF}{dt}$ → infiltration rate at position and time at cumulative inf. has taken place

$\frac{dF}{dt} = f = K_s\left(\frac{h_{pe}(\theta_s - \theta_o)}{F} + 1\right)$

f_c = ... wet potential, total infiltration 2 time to

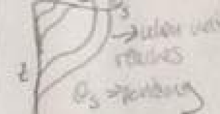
→ given that $P > f$ to reach a depth of .3m → convert $F = .3 \cdot$ ... plug F into G-A → solve 4 t → produce surface runoff @ wet f_c

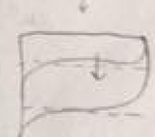
HW Problem → $\theta_s - \theta_o = .2$, $\theta_s = .5$, $K_{sat} = .01$

→ since $f_c > P$, → f_c = function of time, can't read off graph. must solve 4 t from F when $f_c = P$

2 ways to pond →  → $P > f_c$

if $P < f$ @ start → then @ $t > t_p$

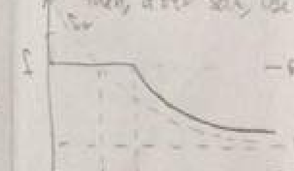




f_c asymptotes to K_s

If $P \leq K_s$, $f = P$

If $P > K_s$, $f = P$ till $F = P\,t_p$; then, after soil, use G-A to get F → f



after ponding move $f_c(t)$ to the right by $t_p - t_s$ = f

if $p > f_c$ then $f = f_c$

$\frac{\partial s}{\partial t} = -\frac{\partial(q\,s)}{\partial x} + i$

→ downslope flux

$\rightarrow q = -K\left(\sin\alpha + \frac{\partial d}{\partial x}\cos\alpha\right)$

$\rightarrow \frac{\partial h}{\partial x} = \sin\alpha + \frac{\partial d}{\partial x}\cos\alpha$

$\Delta S = \Delta h \cdot$ porosity

↳ more accurate is the drainable porosity obtained by experiments + tables

We account $\theta \neq$ Conv 0 to θ_1

$\phi_e \frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_{sat}\left(\sin\alpha + \frac{\partial h}{\partial x}\cos\alpha\right)h\right) + I$

ET: pan evap ~ potential evap [L T^{-1}]

Latent Heat of Evap: J/kg needed 2 evap

Latent Heat Flux: atmospheric flux of water vapor (≈ energy)

energy, humidity ≈ max limit PE, water limit → limits evap

Actual Evap = E = EP f(water avail) A

↳ EP = $K_{pan} \cdot E_{pan}$

↳ 0.75 4 lakes

$EP(L/T) \times A \times \rho \times L$ = energy 2 evap (L/T)

↳ max amt of en 2 evap that depth

If water/limiting, Trans = EP

Trans: $E = EP \times f(\text{water avail}) \times LAI \times$ canopy area

For stem/roots $\rightarrow J = -AK\Delta\Psi$

L^3/T ← conductance (L/T), Δ in water pot (L)

Pot in plants: elev, matric, osmotic

Given: mat pot = -2m, Tree = 10m tall, 5m roots

$A_{roots} = 20m^2$, $K_{st} = 0.01/h$, Evap = 1 m/h

$A_{stem} = .5m^2$, $K_{root} = .1/h$, LAI = 1 m/m

canopy = 10m^2

root p_s = 100 m^2

Find H_2O potent. @ top of roots + top of stem

assume no storage, flux Q roots, root-stem, stem-can, can-atm = equal

4 Eqns: $J = EP \times A_{can} \times LAI$

unknowns

$J = -(\Psi_{can} - \Psi_{stem}) A_{stem} K_{stem}$

$J = -(\Psi_{stem} - \Psi_{root}) A_{root} K_{root}$

$J = -(\Psi_{root} - \Psi_{soil}) A_{root-soil} K_{soil-root}$

Ψ_{soil} = negative when soil is drier

cavitation → bubbles in xylem when too dry → separates water

Given trees = 60m, xylem $r = 10^{-6}$m, surface tension = 7.2×10^{-2} N/m

find: capillary rise

$\rightarrow \Delta P = \rho g \Delta h = \frac{2\sigma}{r} = \rho g h$

Find potentials in way of $\Delta\Psi$ — Simplified Gardner model

$\rightarrow h = \frac{2\sigma}{r\rho g}$, given mat = 3m below w/ matric pot = -2m

Find most potential that drives a flow from bottom of roots 2 canopy (cap rise = elev pot + stem)

$\Psi_c = 10 + \Psi_{c,mat}$

$\Psi_{st} = 3 + \Psi_{s,mat}$

$\Psi_r = 0 + \Psi_{r,mat}$

$\Psi_r = \Psi_{r,mat} + \Psi_{r,mat}$

$\Psi_r = -3m - 2m = -5m$?

For $J \rightarrow \Psi_{st} - \Psi_r < 0 \rightarrow \Psi_c - \Psi_{st} < 0$

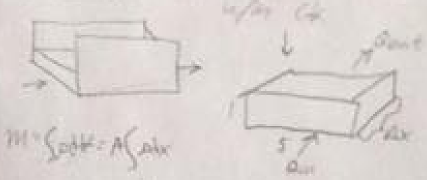
$\Psi_{st} - \Psi_r = \Psi_{s,mat} + 5 < 0 \rightarrow \Psi_{s,mat} = -5m$

$\Psi_{st} - \Psi_c = 60m + \Psi_{c,mat} - .15m + .5m < 0$

$\Psi_{c,mat} = -64.9m$

trees?

Draw CV w/ finite volume of x, derive expression 4 mass balance w/ CV



$m = \int \rho dV = A \int \rho dx$

CV mass: $\frac{dm}{dt} = M_{in} - M_{out}$

$\rightarrow \frac{\partial(S\Delta x\rho)}{\partial t} = S\rho(x)u(x) - S\rho(x+\Delta x)u(x+\Delta x)$

divide by $c^t S\Delta x$

$\frac{\partial}{\partial t}(\bar{\rho}) = -\frac{1}{\Delta x}\left[\rho(x+\Delta x)u(x+\Delta x) - \rho(x)u(x)\right]$

sub $f(x) = \rho(x)u(x)$

$\rightarrow \frac{\partial}{\partial t}(\bar{\rho}) = -\frac{1}{\Delta x}\left[f(x+\Delta x) - f(x)\right] = -\frac{df(x)}{dx}$

$\rightarrow u(x) = c^t \rightarrow \frac{\partial\rho}{\partial t} = -u(x)\frac{d}{dx}(\rho x)$

know $\frac{d}{dx}(\rho(x)) = -0.001\frac{kg}{m^4}$

$\rightarrow \frac{\partial\rho}{\partial x} = 0.001u(x)$

↳ implies that through

$\forall$ is @ SS, ρ is not

→ now clogged w/ silt $\phi = 3$

→ output = 1000 kg, $K_{sat} = 0.5$ m/h

→ find max dh btw $t = 0, 100$ m?

$t = 100 \rightarrow \rho = 900$ kg/m3 (given) $\rightarrow \frac{1000 kg}{h} = \frac{1000}{900}\frac{m^3}{hr}$

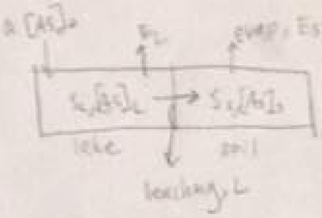
given A, use $Q = -K_{sat} A \frac{dh}{dL}$

→ in porous media w/ changing height

$M_{in} = \rho(x) K_{sat} w h(x) \frac{dh}{dx}|_x$

$e_s = a \exp(b^T/T+c)$ $a = .611$ $b = 17.5$ $c = 240.97°C$

$\frac{ds}{dt} = (P - I) - E - Q -$ other



a) Matric pot drives it

$h_{lake} - h_{soil}$ "upwards" → $h_{soil} < h_{pore}$

$h_{soil} - h_{el} < h_{mat}$

b) water balance

$\frac{dS_L}{dt} = Q - L - E_L$

$\frac{dS_s}{dt} = L - E_s$

c) mass balance

$\frac{d[AS_L]}{dt} = \frac{[AS]_0 Q - [AS_L](L - E_L)}{S_L}$

$\frac{d[AS_s]}{dt} = \frac{[AS_L]L}{S_s} \rightarrow$ no evap (salts don't evap)

d) $E_L = kS_L$ & $Q = c^t = L$

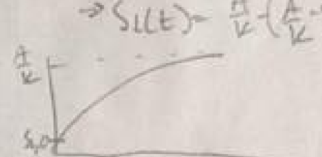
sketch storage in lake as f(t)

$\frac{dS_L}{dt} = Q - L - kS_L$

$\rightarrow \int_{S_{L0}}^{S_L(t)} \frac{dS_L}{Q - L - kS_L} = \int_0^t dt$

$\rightarrow -\frac{1}{k}\ln\left(\frac{A - kS_L(t)}{A - kS_{L0}}\right) = t$

$\rightarrow S_L(t) = \frac{A}{k} - \left(\frac{A}{k} - S_{L,0}\right)e^{-kt}$



Assume $S_L, S_s, \&\ [AS]_L$ = constant in time

solve 4 rate of increase of $[AS]_s$ of $S_L, E_L, E_s, [AS]_0$

$\rightarrow Q - L - E_L = 0 = L - E_s = Q[AS]_0 - L[AS]_L = 0$

$\rightarrow \frac{d[AS]_s}{dt} = \frac{[AS]_L L}{S_s} = \frac{Q}{S_s}\left(\frac{Q}{L}[AS]_0\right) = \frac{E_L + E_s}{S_s} A_{S0}$

$F = \Psi_f(\theta_s - \theta_i)$

$f = K_s\left(h_{mat}\frac{(\theta_s - \theta_i)}{F} + 1\right)$

$F - h_{mat}(\theta_s - \theta_i)\ln\left[1 + \frac{F}{h_{mat}(\theta_s - \theta_i)}\right] = K_{sat} t$