A **Random Variable** X on a sample space Ω is a function that assigns to each sample point

Expectation: E(X) = sum(a * Pr[X=a]) - (mean) 'balance point' like center of gravity

E(X+Y) = E(X) + E(Y) WORKS REGARDLESS OF R.V'sE(cX) = cE(X)

use indicator variables! $X = X_1 + X_2 + ... + X_n$ (1 if yes, 0 if no)

$X = X_1 + X_2 + ... + X_n$ $E(X^2) = E((X_1 + X_2 + ... + X_n)^2) = E(sum(X_i^2)$ $\int_{-\infty}^{\infty} xf(x)dx$))

Variance: expected distance from mean; $Var(X) = E((X - \mu)^2) =$ **$E(X^2) - (E(X))^2$**

$$\int_{-\infty}^{\infty} x^2 f(x)dx - \left(\int_{-\infty}^{\infty} xf(x)dx\right)^2$$

Binomial distribution - X ~ Bin(n,p) BINS? $Pr[X=i] = (n \text{ choose } i)(p^i)(1-p)^{n-i}$, (sum = 1)

flipping coins, X = number of heads E(X) = np (linearity) Var(X) = np(1-p)

applications in probability of receiving packet (error correction, so we can choose k, etc)

Geometric distribution - X~Geom(p) discrete wait time until.. $Pr[X=i] = (1-p)^{i-1}p$

$E(X) = 0p_o+1p_1+2p_2+3p_3+... =$ sum(Pr[X≥i]) = sum($(1-p)^{i-1}$) = 1 / [1 - (1-p)] = 1/p

flipping coins, X = number of tosses until head appears {H, TH, TTH, ...} infinite sample space

Poisson distribution - 'rare events' n is large, constant rate (λ)

n balls into n/λ bins λ=rate Pr[all balls miss bin 1] = $(1 - \lambda/n)^n$ (if n → infinity, Pr → $e^{-\lambda}$)

Pr[X=i] / Pr[X=i-1] → λ/i as n → infinity **$p_i = e^{-\lambda}(\lambda^i/i!)$** (~Taylor expansion, e^x)

E(X) = Var(X) = λ *events =numerous, disjoint, independent, large relative to density of success*

Fixed points (random shuffling): E(X) = 1; Var(X) = 1;

Markov's Inequality - Pr[X ≥ a] ≤ E(X) / a **Non-negative only**

Chebyshev's Inequality - $Pr[|X - \mu| \geq a] \leq Var(X) / (a^2)$

corollary - $Pr[|X - \mu| \geq b\sigma] \leq 1/b^2$

Var(X+Y) = Var(X) + Var(Y); E(XY) = E(X)E(Y) ONLY IF INDEPENDENT

Var(X+Y) = Var(X) + Var(Y) + 2(E(XY) - E(X)E(Y)) **Covariance?**

Independent r.v.: Pr[X=a,Y=b] = Pr[X=a]Pr[Y=b] ∀a,b

indicator r.v.s for independent events are independent

random sampling - P(someone is Democrat) = $A_n = (1/n)S_n$ $S_n = X_1 + ... + X_n$

$E(A_n)$ = p (what we want) $Var(A_n) = \sigma^2/n$

Parameters E (error); d (confidence) $Pr[|A_n - p| \geq Ep] \leq d$how large does n need to be?

Chebyshev: $d \leq (Var(A_n))/(E)^2$ (same bound); **$n \geq \sigma^2/(E^2d)$**; (max σ^2=p(1-p)=0.25); **$n \geq 1/(4E^2d)$**

General expectation: relative error, solving for eμ

$Var(A_n) = \sigma^2/n$ **$d \leq (Var(A_n))/(E\mu)^2$** **$n \geq \sigma^2/(\mu^2E^2d)$** $Y = \frac{X - \mu}{\sigma}$

f(x) ~ probability per unit length Var(X) =

Joint Density - double integral; ~ probability per unit area.

X,Y independent if a<X<b and c<Y<d independent ∀a,b,c,d; f(x,y) = f(x)f(y)

Exponential distribution - analog to geometric; wait for continuous time; λ = rate of success

f(x) = { $\lambda e^{-\lambda x}; x \geq 0$ 0 otherwise} E(X) = 1/lambda Var(X) = $1/(\text{lambda})^2$

Pr[X > t] = $e^{-\lambda t}$ geometric: $Pr[X > kd] = (1-p)^k = (1- \lambda d)^k$ d = rate of trials

Bijections, Injections, Surjections bijection = mapping, injective and surjective.

INFINITE SETS? **countable if |set| ≤ |N|** N maps to Z+, N maps to Z

N maps to Q ($|N| \leq |Q|$ and $|Q| \leq |N^2| = |N|$), N maps to all binary strings, all polynomials

Cantor-Bernstein theorem = one-to-one functions, both ways implies same cardinality

Cantor's Diagonalization = list of all real numbers, diagonal = real number, add 1 to every number of diagonal, this new number must not exist in list. Thus, reals are uncountable.