

HOW TO OPTIMIZE THAT SHIT!

Find projection z of x onto $\{x_0 + tu : t \in \mathbb{R}\}$

$$\min_t \|x - x_0 - tu\|_2^2 \rightarrow t^* = u^T(x - x_0)$$

$$z = x_0 + t^*u$$

For hyperplane $H = \{x : a^T x = b\}$

if $\|a\|_2 = 1$, projection of 0 on H is ba ,

solving $\min_x \|x\|_2 : x \in H$

For affine set $\{x : Ax = b\}$

$$x^* = A^T(AA^T)^{-1}b$$

Linear programs:

General form: $\min_x c^T x + d : Gx \leq h, Ax = b$

Standard form: $\min_x c^T x : Ax = b, x \geq 0$

Tricks for standardization:

① $\max_x c^T x \leftrightarrow \min_x (-c)^T x$

② $Ax \leq b \leftrightarrow Ax + s = b, s \geq 0$

③ If x is unrestricted in sign,
 - introduce $x^+, x^- \geq 0$
 - replace x by $(x^+ - x^-)$

Quadratic Programs:

Standard form: $(H \succeq 0)$

$$\min_x \frac{1}{2} x^T H x + c^T x : Gx = h, Ax \leq b$$

Constrained LS: $\min_x \|Rx - y\|_2^2 : Gx = h, Ax \leq b$

transforms to standard with $H = 2R^T R \succeq 0, c^T = -2y^T R$

Least squares:

$$\min_x \|Ax - y\|_2^2 \quad x^* = (A^T A)^{-1} A^T y$$

Variants:

$$\min_x \|Ax - y\|_2^2 : (x = d)$$

$$\min_x \|Ax - y\|_2^2 + \lambda \|x\|_2^2$$

$$\rightarrow \min_z \|\tilde{A}z - \tilde{y}\|_2^2$$

$$\rightarrow \min_x \|\tilde{A}x - \tilde{y}\|_2^2, \tilde{A} = \begin{bmatrix} A \\ \sqrt{\lambda} I \end{bmatrix}, \tilde{y} = \begin{bmatrix} y \\ 0 \end{bmatrix}$$

where $\tilde{A} = AN, \tilde{y} = y - Ax_0$

N spans $\mathcal{N}(C)$, x_0 solves $(x = d)$

$$\min_x \|Ax - y\|_2^2 + x^T W x, W \succ 0$$

$$x^* = (A^T A + W)^{-1} A^T y$$

$$\min_x \|x\|_2 : Ax = y$$

if A is full row rank, $x^* = A^T(AA^T)^{-1}y$

SOCP:

Standard form:

$$\min_x c^T x : \|A_i x + b_i\|_2 \leq c_i^T x + d_i$$

rotated SOC:

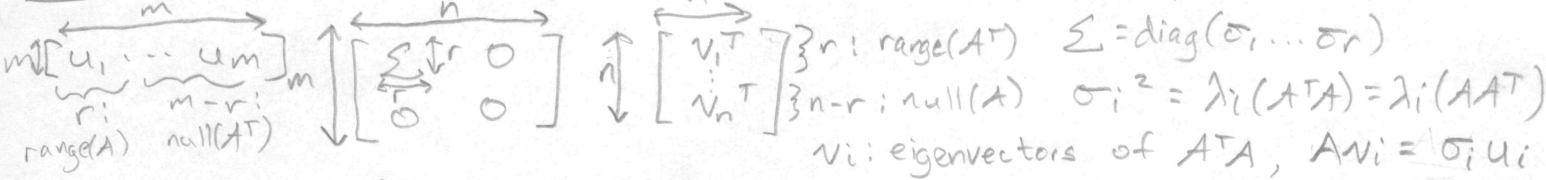
$$\|x\|_2^2 \leq yz \iff \left\| \begin{pmatrix} 2x \\ y-z \end{pmatrix} \right\|_2 \leq y+z$$

for $y, z \geq 0$

$$x^T Q x + c^T x \leq t \iff w^T w \leq y, w = Q^{1/2} x, y = t - c^T x$$

$$Q \succeq 0$$

SVD: $A = U \Sigma V^T, A \in \mathbb{R}^{m \times n}$



$\Sigma = \text{diag}(\sigma_1, \dots, \sigma_r)$
 $\sigma_i^2 = \lambda_i(A^T A) = \lambda_i(A A^T)$
 v_i : eigenvectors of $A^T A, A v_i = \sigma_i u_i$

$\|A\|_F^2 = \text{Tr}(A^T A) = \sum_i \sigma_i^2$

$\mathcal{E}(A) = \{Ax : \|x\|_2 \leq 1\}$

sum of dyads: $A = \sum_i \sigma_i u_i v_i^T$ U maps $\mathcal{E}(\hat{\Sigma}) \rightarrow \mathcal{E}(A)$

$\text{Tr}(A^T A) = \text{Tr}(V \hat{\Sigma}^T U^T U \hat{\Sigma} V^T) = \text{Tr}(V \hat{\Sigma}^T \hat{\Sigma} V^T) = \text{Tr}(\hat{\Sigma}^2)$