SEMICONDUCTOR PROPERTIES

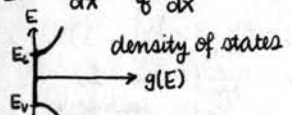
* materials * crystal structure * e^- & h^+ * energy bands * density of states * Fermi-Dirac * thermal equilibrium * carriers * drift & diffusion * RG * diffusion equations * quasi-Fermi levels

Important quantities

- $q = 1.6\times10^{-19}$ C
- $\varepsilon_0 = 8.854\times10^{-14}$ F/cm
- $k_B = 8.62\times10^{-5}$ eV/K
- $h = 4.14\times10^{-5}$ eV·s
- $m_e = 9.1\times10^{-31}$ kg
- $\frac{k_BT}{q} = 26$ mV = 0.026V
- $k_BT\ln(10) = 60$ meV = 0.060 eV
- 1eV = 1.6×10^{-19} J
- $a_0(Si) = 0.543$ nm
- $n_i(Si) = 10^{10}\ cm^{-3}$ (T=300K)

Energy band

- $E_G \equiv E_C - E_V$
  - ↳ $E_G(Si) = 1.12$ eV
  - ↳ $E_G(SiO_2) = 9$ eV
- $U = -qV$
- $\vec{E} = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx}$
- density of states



$g_c(E) = \frac{8\pi\sqrt{2}}{h^3}(m_{n,DOS})^{3/2}(E-E_c)^{1/2}$

$g_v(E) = \frac{8\pi\sqrt{2}}{h^3}(m_{p,DOS})^{3/2}(E_v-E)^{1/2}$

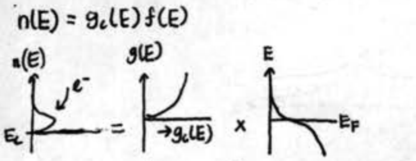
Doping

- Donors (V): P, As, Sb ↔ N_D
- Acceptors (III): B, Al, Ga ↔ N_A
- ionisation energy ↓ ⇒ good
- $np = n_i^2$ (thermal equilibrium)
- $n = \frac{1}{2}(N_D - N_A) + \left[\left(\frac{N_D-N_A}{2}\right)^2 + n_i^2\right]^{1/2}$
- $p = \frac{1}{2}(N_A - N_D) + \left[\left(\frac{N_A-N_D}{2}\right)^2 + n_i^2\right]^{1/2}$

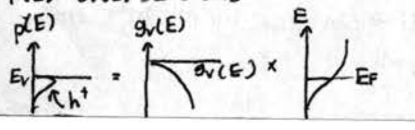
Fermi-Dirac, Boltzmann

- $f(E) = \frac{1}{1 + e^{\frac{E-E_F}{k_BT}}}$
- equilibrium ⇒ E_F constant with x
- Boltzmann approximation
  - $E - E_F > 3k_BT$: $f(E) \approx e^{-\frac{(E-E_F)}{k_BT}}$
  - $E_F - E > 3k_BT$: $1 - f(E) \approx 1 - e^{-\frac{(E-E_F)}{k_BT}}$
- equilibrium carrier concentration
  - $n \approx \int_{E_c}^{\infty} n(E)dE = \int_{E_c}^{\infty} g_c(E)f(E)dE$
  - Boltz approx: $n = N_c e^{-\frac{(E_c-E_F)}{k_BT}}$, $N_c = 2\left(\frac{2\pi m_{n,DOS}k_BT}{h^2}\right)^{3/2}$
  - $p \approx \int_{-\infty}^{E_v} p(E)dE = \int_{-\infty}^{E_v} g_v(E)[1-f(E)]dE$
  - $= N_v e^{-\frac{(E_F-E_v)}{k_BT}}$, $N_v = 2\left(\frac{2\pi m_{p,DOS}k_BT}{h^2}\right)^{3/2}$

$n(E) = g_c(E)f(E)$



$p(E) = g_v(E)[1 - f(E)]$



Fermi-Dirac, Thermal Equilibrium

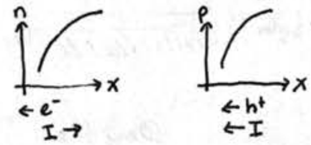
- $np = n_i^2 \Rightarrow n_i = (N_cN_v)^{1/2} e^{-\frac{E_G}{2k_BT}}$
- $N_c(Si) = 2.82\times10^{19}\ cm^{-3}$
- $N_v(Si) = 1.83\times10^{19}\ cm^{-3}$ } T=300K
- intrinsic semiconductor: $n = p = n_i$, $E_F = E_i$
  - ↳ $n_0 = n_i e^{\frac{E_F-E_i}{k_BT}}$
  - ↳ $p = n_i e^{\frac{E_i-E_F}{k_BT}}$
  - ↳ $E_i = \frac{1}{2}(E_c + E_v) + \frac{1}{2}k_BT\ln\left(\frac{N_v}{N_c}\right)$ [$= \frac{3}{4}k_BT\ln\left(\frac{m_p^*}{m_n^*}\right)$]
- extrinsic semiconductor w/ degenerate doping ($N > 10^{18}\ cm^{-3}$)
  - ↳ p+: $E_F \approx E_v$
  - ↳ n+: $E_F \approx E_c$
  - ↳ $\Delta E_G \approx 3.5\times10^{-8} N^{1/3}\left(\frac{300}{T}\right)$ meV
    - = 75 meV ($N = 10^{19}\ cm^{-3}$)
    - = 35 meV ($N = 10^{18}\ cm^{-3}$)
- $E_F = E_C - k_BT\ln\left(\frac{N_c}{n}\right)$
- $E_F = E_v + k_BT\ln\left(\frac{N_v}{p}\right)$

Mobile carrier action in semiconductor

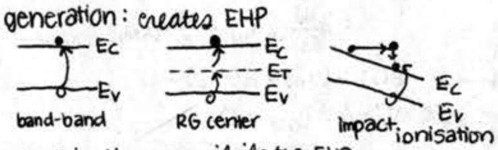
- DRIFT: due to $\vec{E}$
- DIFFUSION: due to [carrier] or T gradient
- RG: EHPs created/annihilated
- effective mass: due to acceleration under $\vec{E}$
  - $m_n^* = 0.26m_0$
  - $m_p^* = 0.39m_0$ } $m_0 = 9.1\times10^{-31}$ kg
- kinetic energy & thermal velocity
  - $T = \frac{3}{2}k_BT = \frac{1}{2}m_n^* v_{th}^2$
  - $v_{th} = \left(\frac{3k_BT}{m_n^*}\right)^{1/2} \approx 2.3\times10^7$ cm/s
- $\vec{I}$ direction opposite to $\vec{E}$ (e^-)
- large $\vec{E} \Rightarrow v_{drift}$ saturates → $v_{drift,max}$
- carrier mobility
  - $\mu_n = q\frac{\tau_{mn}}{m_n^*}$ ← average time between scattering events
  - $\mu_p = q\frac{\tau_{mp}}{m_p^*}$
  - $\vec{v}_{drift} = \mu\vec{E}$
- mean free path: $l = v_{th}\tau_{mp/n}$ — average distance between collisions
- carrier scattering
  - phonon scattering: $\mu_{phon} \propto \frac{1}{T^{3/2}}$
  - dopant scattering: $\mu_{dop} \propto T^{3/2}$
- $\frac{1}{\mu} = \frac{1}{\mu_{phon}} + \frac{1}{\mu_{dop}}$
- $\frac{1}{\tau} = \frac{1}{\tau_{phon}} + \frac{1}{\tau_{dop}}$
- $J_{drift} = (qp\mu_p + qn\mu_n)E$
  - $\sigma \equiv qp\mu_p + qn\mu_n$
  - $\rho = \frac{1}{\sigma}$
    - $\approx \frac{1}{qp\mu_p}$ (n-type)
    - $\approx \frac{1}{qn\mu_n}$ (p-type)
- $R = \frac{V}{I} = \rho\frac{L}{Wt}$

Carrier diffusion

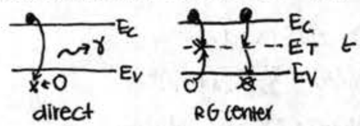
- current density
  - $J_n = J_{n,drift} + J_{n,diff} = qn\mu_n\vec{E} + qD_n\frac{dn}{dx}$
  - $J_p = J_{p,drift} + J_{p,diff} = qp\mu_p\vec{E} + qD_p\frac{dp}{dx}$



- ratio of $n(x) : p(x) \propto \Delta V$ between the points
  - $\Delta V = \frac{1}{q}(E_{i1} - E_{i2}) = \frac{k_BT}{q}\ln\left(\frac{n_2}{n_1}\right)$
- non-uniformly doped semiconductor: $E_c = E_c(x)$ and $E_v = E_v(x)$ so as to keep E_F constant throughout semiconductor
  - $n = N_c e^{-\frac{(E_c-E_F)}{k_BT}} \Rightarrow \frac{dn}{dx} = -\frac{n}{k_BT}\frac{dE_c}{dx} = -\frac{n}{k_BT}q\mathcal{E}$
  - $p = N_v e^{-\frac{(E_F-E_v)}{k_BT}} \Rightarrow \frac{dp}{dx} = +\frac{p}{k_BT}\frac{dE_v}{dx} = +\frac{p}{k_BT}q\mathcal{E}$
- equilibrium: $J_p = J_n = 0 \Rightarrow J_{drift} = -J_{diff}$
- $D = \mu\frac{k_BT}{q}$ (diffusion constant)
- quasi-neutrality approximation
  - $N_D(x) + p(x) = N_A(x) + n(x)$
  - $n(x) \approx N_D(x) - N_A(x)$ (n-type)
  - $p(x) \approx N_A(x) - N_D(x)$ (p-type)
- generation: creates EHP



- recombination: annihilates EHP



RG center ← primary recombination mechanism in Si

- low level injection: majority carrier concentrations not significantly affected by disturbance from equilibrium
  - $n \approx n_0$ (n-type)
  - $p \approx p_0$ (p-type)
- minority carrier lifetime: average time that minority carrier survives in a sea of majority carriers before recombination
  - $\tau_{p/n} \in [10^{-9}, 10^{-6}]$ for Si
  - depends on N: deep trap energy states facilitate RG in RG center
  - sudden injection of excess carriers → system relaxes back to equilibrium via RG: $\frac{\partial n}{\partial t} = -\frac{\Delta n}{\tau_n}$, $\frac{\partial p}{\partial t} = -\frac{\Delta p}{\tau_p}$
- $-\frac{\partial\Delta n}{\partial t} = -\frac{\partial\Delta p}{\partial t} = \frac{np - n_i^2}{\tau_p(n+n_1) + \tau_n(p+p_1)}$, $n_1 | p_1 = n_i e^{\frac{(E_T-E_i)}{k_BT}} / n_i e^{\frac{(E_i-E_T)}{k_BT}}$

Minority carrier diffusion equation

- ASSUMPTIONS
  - (1) Small $\vec{E}$: $J = J_{drift} + J_{diff} \approx J_{diff}$ ($J_{drift} \propto \vec{E}$)
  - (2) Uniform doping: n_0, p_0 independent of x
  - (3) LLI
- $\frac{\partial\Delta n_p}{\partial t} = D_n\frac{\partial^2\Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$
- $\frac{\partial\Delta p_n}{\partial t} = D_p\frac{\partial^2\Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$
  - steady state: $\frac{\partial\Delta n_p}{\partial t} = \frac{\partial\Delta p_n}{\partial t} = 0$
  - $J_{diff} = 0$: $D_n\frac{\partial^2\Delta n_p}{\partial x^2} = D_p\frac{\partial^2\Delta p_n}{\partial x^2} = 0$
  - no RG: $\frac{\Delta n_p}{\tau_n} = \frac{\Delta p_n}{\tau_p} = 0$
  - no γ: $G_L = 0$
- boundary conditions: $\Delta p_n(0) = \Delta p_{n_0}$, $\Delta p_n(\infty) = 0$
  - $\Delta n_p(0) = \Delta n_{p_0}$, $\Delta n_p(-\infty) = 0$

Quasi-Fermi level: $np \neq n_i^2$ when $\Delta n / \Delta p \neq 0$:

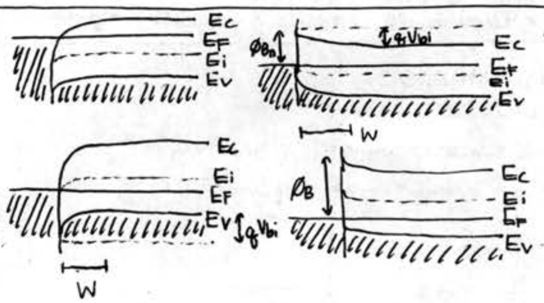
- $n = n_i e^{\frac{F_N - E_i}{k_BT}} \Rightarrow F_N = k_BT\ln\left(\frac{n}{n_i}\right) + E_i$
- $p = n_i e^{\frac{E_i - F_P}{k_BT}} \Rightarrow F_P = E_i - k_BT\ln\left(\frac{p}{n_i}\right)$

METAL-SEMICONDUCTOR CONTACTS

* work function
* energy band diagrams
* depletion width
* small signal C
* Schottky diode
* practical ohmic contacts

n-type

p-type



$\phi_M < \phi_S$ (Ohmic)

$\phi_M > \phi_S$ (rectifying)

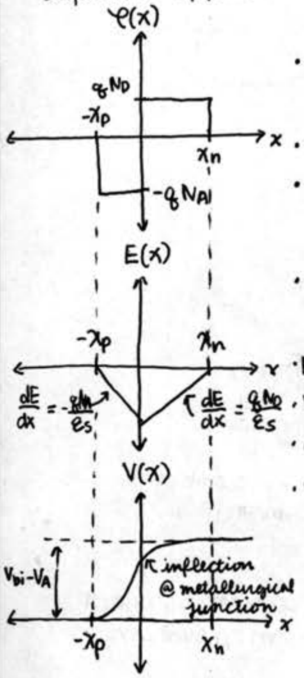
- $\phi_{Bn} = \phi_M - \chi \Rightarrow \phi_{bi} = \phi_{Bn} - (E_C - E_F)_{FB}$ (n-type)
- $\phi_{Bp} = \chi + E_G - \phi_M \Rightarrow \phi_{bi} = \phi_{Bp} - (E_F - E_V)_{FB}$ (p-type)
- $|v_x| \geq \left[\frac{2q}{m_n^*}(V_{bi} - V_A)\right]^{1/2}$ (minimum velocity for e^- to cross junction into metal)
- $I_{S\to M} = AJ_S e^{\frac{qV_A}{k_BT}}$, $J_S \equiv 120 \frac{m_n^*}{m_0} T^2 e^{-\frac{\phi_B}{k_BT}}$ [A/cm²]
- $I_{M\to S} = -I_{S\to M}(V_A = 0)$
- $I = AJ_S(e^{\frac{qV_A}{k_BT}} - 1)$ [schottky diode] ← preferred for low V, high I
- tunnelling current density
  - $P = e^{-H(\phi_{Bn} - V_A)/\sqrt{N_D}}$, $H \approx 5.4 \times 10^9 \sqrt{\frac{m_n^*}{m_0}}$ cm$^{-3/2}$V^{-1}
  - $J_{S\to M} = qN_D\sqrt{\frac{k_BT}{2\pi m_n^*}}\, e^{-H(\phi_{Bn} - V_A)/\sqrt{N_D}}$

pn JUNCTIONS

* electrostatics
* ideal diode I-V
* narrow base diode
* breakdown
* deviations from ideal I-V
* charge control model
* small signal
* transient response
* applications

Electrostatics

- Depletion approximation

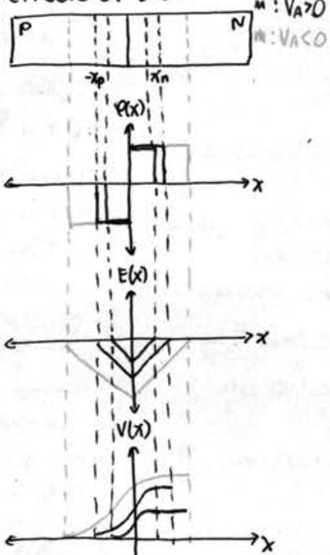


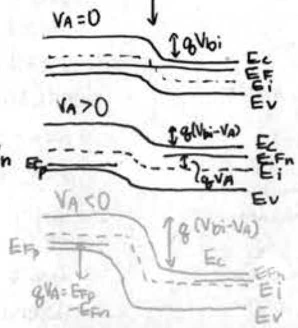
- $\rho(x) = \begin{cases} qN_D, & 0 \leq x \leq x_n \\ -qN_A, & -x_p \leq x \leq 0 \\ 0, & \text{else} \end{cases}$
- $qN_D x_n = qN_A x_p$
- depletion length larger for less doped side
- $E(x) = \int \frac{\rho(x)}{\varepsilon_s} dx = \frac{qN_D}{\varepsilon_s}(x - x_n) - \frac{qN_A}{\varepsilon_s}(x + x_p)$
- BC: $E(x_n) = E(-x_p) = 0$
- $V_A = 0$: $I_{net} = 0$
- counteracts diffusion of carriers
- $V(x) = -\int E(x)dx$
  - ↳ $V_p(x) = \frac{qN_A}{2\varepsilon_s}(x + x_p)^2$
  - ↳ $V_n(x) = -\frac{qN_D}{2\varepsilon_s}(x_n - x)^2 + V_{bi}$

- depletion width: use $V_n(x=0) = V_p(x=0)$, $N_A x_p = N_D x_n$
  - ↳ $x_p = \left[\frac{2\varepsilon_s(V_{bi} - V_A)}{q}\left(\frac{N_D}{N_A}\frac{1}{N_D + N_A}\right)\right]^{1/2}$
  - ↳ $x_n = \left[\frac{2\varepsilon_s(V_{bi} - V_A)}{q}\left(\frac{N_A}{N_D}\frac{1}{N_D + N_A}\right)\right]^{1/2}$
  - ↳ $W = x_p + x_n = \left[\frac{2\varepsilon_s(V_{bi} - V_A)}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\right]^{1/2}$
    - * $W \approx x_n$ (p+n)
    - * $W \approx x_p$ (pn+)
    - * $\propto \sqrt{V_{bi}-V_A}$, $\frac{1}{N_D}$ or $\frac{1}{N_A}$
- peak $\vec{E}$: $|E(0)| = \frac{2(V_{bi} - V_A)}{W} = \left[\frac{2qN(V_{bi} - V_A)}{\varepsilon_s}\right]^{1/2}$
- built-in potential V_{bi}
  - ↳ $\phi_{bi} = qV_{bi} = (E_i - E_F)_p + (E_F - E_i)_n$
  - ↳ $(E_i - E_F)_p = \begin{cases} \frac{1}{2}E_G, & \text{degenerate p} \\ k_BT\ln\left(\frac{N_A}{n_i}\right), & \text{non-degenerate p} \end{cases}$
  - ↳ $(E_F - E_i)_n = \begin{cases} \frac{1}{2}E_G, & \text{degenerate n} \\ k_BT\ln\left(\frac{N_D}{n_i}\right), & \text{non-degenerate n} \end{cases}$
  - ↳ $V_{bi} = \frac{k_BT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$, both sides non-deg. doped
  - ↳ $(E_C - E_F)_{FB} = \frac{1}{2}E_G - (E_F - E_i)$

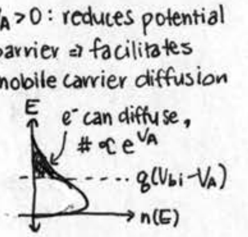
Electrostatics (cont.)

- effects of bias





$V_A > 0$: reduces potential barrier ⇒ facilitates mobile carrier diffusion



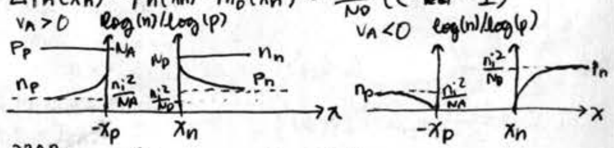
$V_A < 0$: increases potential barrier ⇒ any minority carrier that happens to diffuse is collected into QNR ⇒ I<0

$V_A > 0$: minority carriers INJECTED

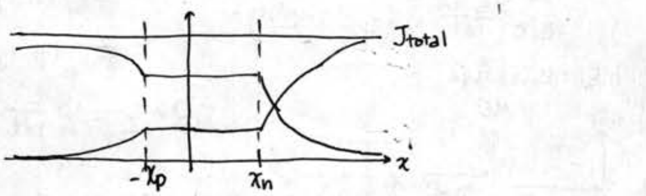
$V_A < 0$: minority carriers COLLECTED

Ideal Diode Equation

- assumptions & conditions
  - uniformly doped step junction
  - steady state: $\frac{\partial^2 \Delta p_n(x)}{\partial x^2} = \frac{\partial^2 \Delta n_p(x)}{\partial x^2} = 0$
  - LLI in QNR ⇒ law of junction
  - negligible RG in DP: $\frac{dJ_n}{dx} = \frac{dJ_p}{dx} = 0$
- (1) Obtain $\Delta n_p(x, V_A)$, $\Delta p_n(x, V_A)$
  - $pn = n_i^2 e^{\frac{qV_A}{k_BT}}$
  - $\Delta n_p(-x_p) = n_p(-x_p) - n_{p0}(-x_p) = \frac{n_i^2}{N_A}(e^{\frac{qV_A}{k_BT}} - 1)$
  - $\Delta p_n(x_n) = p_n(x_n) - p_{n0}(x_n) = \frac{n_i^2}{N_D}(e^{\frac{qV_A}{k_BT}} - 1)$



  - $\frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{L_p^2}$, $L_p \equiv (D_p \tau_p)^{1/2}$
    - ↳ $\Delta p_n(x_n) = \frac{n_i^2}{N_D}(e^{\frac{qV_A}{k_BT}} - 1)$, $\Delta p_n(\infty) \to 0$
    - ↳ $\Delta p_n(x') = \frac{n_i^2}{N_D}(e^{\frac{qV_A}{k_BT}} - 1)e^{-\frac{x'}{L_p}}$
  - $\frac{\partial^2 \Delta n_p}{\partial x^2} = \frac{\Delta n_p}{L_n^2}$, $L_n = (D_n \tau_n)^{1/2}$
    - ↳ $\Delta n_p(-x_p) = \frac{n_i^2}{N_A}(e^{\frac{qV_A}{k_BT}} - 1)$, $\Delta n_p(-\infty) \to 0$
    - ↳ $\Delta n_p(x'') = \frac{n_i^2}{N_A}(e^{\frac{qV_A}{k_BT}} - 1)e^{-\frac{x''}{L_n}}$
- (2) Determine J_n, J_p to find total J
  - $J_n = -qD_n \frac{\partial \Delta n_p(x'')}{\partial x''} = q\frac{D_n}{L_n}\frac{n_i^2}{N_A}(e^{\frac{qV_A}{k_BT}} - 1)e^{-\frac{x''}{L_n}}$
  - $J_p = -qD_p \frac{\partial \Delta p_n(x')}{\partial x'} = q\frac{D_p}{L_p}\frac{n_i^2}{N_D}(e^{\frac{qV_A}{k_BT}} - 1)e^{-\frac{x'}{L_p}}$
  - $J = J_n|_{x''=0} + J_p|_{x'=0}$ (edges of QNR)
  - $J = qn_i^2\left(\frac{D_n}{L_n N_A} + \frac{D_p}{L_p N_D}\right)(e^{\frac{qV_A}{k_BT}} - 1)$
- (3) I-V !!
  - $I = I_0(e^{\frac{qV_A}{k_BT}} - 1)$, $I_0 = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$
    - ↳ $I_0 \approx Aqn_i^2\left(\frac{D_p}{L_p N_D}\right)$ ($N_A \gg N_D$, p-type)
    - ↳ $I_0 \approx Aqn_i^2\left(\frac{D_n}{L_n N_A}\right)$ ($N_D \gg N_A$, n-type)



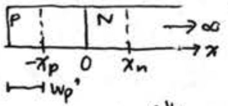
- $J_{diff} \propto \nabla([\text{carrier}])$
- $J_{drift} \propto |[\text{carrier}]|$ ⇒ can ignore for minority carrier
- injection: look for

pn JUNCTIONS (cont.)

*narrow base diode *breakdown *deviations from ideal I-V *charge control model *small signal *transient response

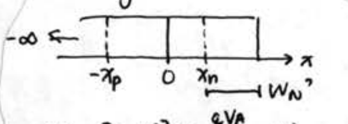
Ideal Diode Equation (cont.)

- Narrow base: W_p', W_n' L_p, L_N
  - BC: $\Delta P_n(x'=0) = \frac{n_i^2}{N_D}(e^{\frac{qV_A}{k_BT}}-1)$
    - $\Delta P_n(x'=x_c') = 0$
    - $\Delta n_p(x''=0) = \frac{n_i^2}{N_A}(e^{\frac{qV_A}{k_BT}}-1)$
    - $\Delta n_p(x''=-x_d'') = 0$
  - P = narrow, n = long



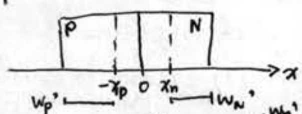
$$J = q\frac{D_n}{L_n}\frac{n_i^2}{N_A}(e^{\frac{qV_A}{k_BT}}-1)\frac{\cosh(\frac{W_p'}{L_n})}{\sinh(\frac{W_p'}{L_n})} + q\frac{D_p}{L_p}\frac{n_i^2}{N_D}(e^{\frac{qV_A}{k_BT}}-1)$$

- P = long, n = narrow



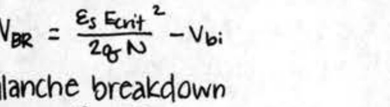
$$J = q\frac{D_n}{L_n}\frac{n_i^2}{N_A}(e^{\frac{qV_A}{k_BT}}-1) + q\frac{D_p}{L_p}\frac{n_i^2}{N_D}(e^{\frac{qV_A}{k_BT}}-1)\frac{\cosh(\frac{W_n'}{L_p})}{\sinh(\frac{W_n'}{L_p})}$$

- p = narrow, n = narrow



$$J = q\frac{D_n}{L_n}\frac{n_i^2}{N_A}(e^{\frac{qV_A}{k_BT}}-1)\frac{\cosh(\frac{W_p'}{L_n})}{\sinh(\frac{W_p'}{L_n})} + q\frac{D_p}{L_p}\frac{n_i^2}{N_D}(e^{\frac{qV_A}{k_BT}}-1)\frac{\cosh(\frac{W_n'}{L_p})}{\sinh(\frac{W_n'}{L_p})}$$

- $W_N' \ll L_p$, $W_p' \gg L_N$ (very narrow)
  - ↳ $\cosh(x) \to 1 + x^2$ as $x \to 0$
  - ↳ $\sinh(x) \to x$ as $x \to 0$
  - ↳ $I = qAn_i^2\left(\frac{D_p}{W_n'N_D} + \frac{D_n}{W_p'N_A}\right)(e^{\frac{qV_A}{k_BT}}-1)$
- narrow base ⇒ negligible recombination
  - ↳ $\Delta n_p / \Delta P_n$ linear
  - ↳ constant

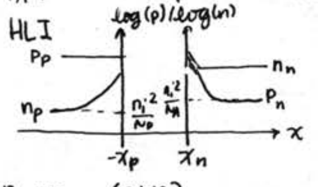


Junction breakdown

- If $(-V_A)$ so large that $E_{max} = |E(0)| > E_{crit}$, then breakdown occurs ← lighter dopant
  - $E_{crit} = \left[\frac{2q(V_{bi}+V_{BR})N}{\varepsilon_s}\right]^{1/2}$
  - $V_{BR} = \frac{\varepsilon_s E_{crit}^2}{2qN} - V_{bi}$
- Avalanche breakdown
  - $N < 10^{18}\ cm^{-3}$
  - $V_{BR} \approx \frac{\varepsilon_s E_{crit}^2}{2qN}$ if $V_{BR} \gg V_{bi}$
  - $V_{BR} \propto T$ since $l \downarrow$
- Tunnelling breakdown
  - $N > 10^{18}\ cm^{-3}$
  - $E_{crit} \approx 10^6\ V/cm$
  - $V_{BR} < 5V$ (Zener) $\propto \frac{1}{T}$ since e^- flux available for tunnelling ↑

Non-ideal Diode

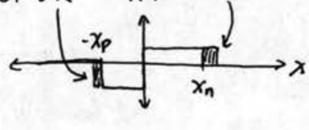
- RG in depletion region
  - contributes additional component of current: $I_{RG} = -qA\int_{-x_p}^{x_n}\frac{\partial p}{\partial t}dx$
  - $V_A < 0$: net GENERATION
    - ↳ $I_{RG} = -\frac{qAn_iW}{2\tau_0}$, $n_i e^{\frac{E_T-E_i}{k_BT}}$, $n_i e^{\frac{E_i-E_T}{k_BT}}$
    - $\tau_0 \equiv \frac{1}{2}(\tau_p\frac{n_1}{n_i} + \tau_n\frac{p_1}{n_i})$
  - $V_A > 0$: net RECOMBINATION
    - ↳ $I_{RG} \propto qAn_iWe^{\frac{qV_A}{2k_BT}}$
- High level Injection (HLI)
  - $V_A \uparrow$ ⇒ less doped side reaches HLI



  - $n_n > n_{n_0}$ (p+n)
  - $P_p > P_{p0}$ (pn+)
  - creates large gradient in majority carrier profile
- series resistance R_s limits increases in current with increasing $V_A > 0$

Charge Control Model

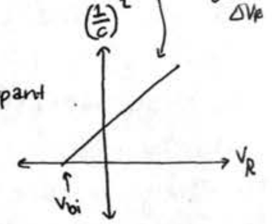
- $V_A > 0$: excess minority carriers stored in QNR
  - $Q_N = -qA\Delta n_p(-x_p)L_N$
  - $Q_P = qA\Delta P_n(x_n)L_p$
- long base diode
  - $I_N(-x_p) = -\frac{Q_N}{\tau_n}$
  - $I_P(x_n) = \frac{Q_P}{\tau_p}$
- narrow base diode
  - $\tau_{tr,n} = \frac{(W_p')^2}{2D_N}$ (e^- in narrow p)
  - $\tau_{tr,p} = \frac{(W_n')^2}{2D_P}$ (h^+ in narrow n)
- stored charge
  - $Q_N = \begin{cases} -qA\frac{n_i^2}{N_A}(e^{\frac{qV_A}{k_BT}}-1)L_N, & \text{long} \\ -qA\frac{n_i^2}{N_A}(e^{\frac{qV_A}{k_BT}}-1)\frac{W_p'}{2}, & \text{narrow} \end{cases}$
  - $Q_P = \begin{cases} qA\frac{n_i^2}{N_D}(e^{\frac{qV_A}{k_BT}}-1)L_p, & \text{long} \\ qA\frac{n_i^2}{N_D}(e^{\frac{qV_A}{k_BT}}-1)\frac{W_n'}{2}, & \text{narrow} \end{cases}$
- steady state diode current: charge supply required to compensate for charge loss via...
  - *recombination in RG (long)*
  - *collection at contact (narrow)*
- excess minority carriers stored in QNR
- majority carries stored at edges of DR



Small signal model

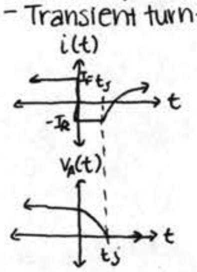
- $I = \frac{V_{AC}}{R} + C\frac{dV_{AC}}{dt}$
- $\frac{1}{R} = \frac{dI_{DC}}{dV_A} = \frac{d}{dV_A}[I_0(e^{\frac{qV_A}{k_BT}}-1)] \approx \frac{d}{dV_A}I_0e^{\frac{qV_A}{k_BT}}$
- $G \equiv \frac{1}{R} = \frac{q}{k_BT}I_0e^{\frac{qV_A}{k_BT}} \approx \frac{I_{DC}}{k_BT/q}$
- depletion Capacitance: due to variation of Q_{dep}
  - $C_J \equiv \left|\frac{dQ_{dep}}{dV_A}\right|$
- diffusion capacitance: due to variation of stored Q_N, Q_P in QNR
  - $C_D \equiv \left|\frac{dQ}{dV_A}\right|$
- one-sided junction: $Q = Q_N + Q_P \approx Q_N$ OR Q_P
  - $C_p = \left|\frac{dQ_p}{dV_A}\right| = \tau_p\frac{dI}{dV_A} = \tau_p G = \tau_p\frac{I_{DC}}{k_BT/q}$
    $= \tau_n\frac{dI}{dV_A} = \tau_n G = \tau_n\frac{I_{DC}}{k_BT/q}$
  - $C_J = \varepsilon_s\frac{A}{W} \to \propto A$, N_A/N_D, $V_A < 0$
  - $C = C_D + C_J$ ← dominates at low V_A, $V_A<0$; C_D dominates at med-high $V_A > 0$
  - $\frac{1}{C_J^2} = \frac{W^2}{A^2\varepsilon_s^2} \approx \frac{2(V_{bi}-V_A)}{A^2q\varepsilon_sN}$ ← lighter dopant
    - ↳ slope $\propto \frac{1}{N}$
    - ↳ x-int = V_{bi}
    - ↳ $V_{bi} = \frac{k_BT}{q}\ln\left(\frac{N_hN_l}{n_i^2}\right)$

$N = \frac{2}{\varepsilon q\frac{\Delta(\frac{1}{C})^2}{\Delta V_R}}$

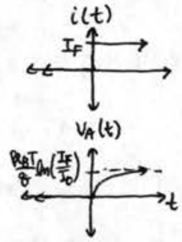


Transient response

- Due to $C_D = \left|\frac{dQ}{dV_A}\right|$, voltage across junction DR can't be changed instantaneously from sudden V_A shut-off
- Transient turn-off



  - to completely shut off diode, ΔP_n & Δn_p must be removed from QNR via net carrier flow or recombination
  - $t_s \approx \tau_p\ln(1+\frac{I_F}{I_R})$ (p+n)
    - ↳ $\propto I_F$ since $Q_P(t=0)$ is larger
    - ↳ $\propto \frac{1}{I_R}$ since rate of h^+ removal increases
    - ↳ $\propto \frac{1}{\tau_p}$ since h^+ annilated faster
- Transient turn-on



  - $v_A(t) = \frac{k_BT}{q}\ln\left[1 + \frac{I_F}{I_0}(1-e^{-\frac{t}{\tau_p}})\right]$
  - $\tau_p \gg$ ⇒ no RG ⇒ turn on time
    - $t_s \approx \frac{\Delta Q}{I_F}$, $\Delta Q = \Delta Q_P + \Delta Q_j$ (excess minority storage in QNR; majority storage at DR edge)

Varactor diode

- Reverse-biased: $-V_A = V_r$
- V-controlled C: $C_J \propto V_r^{-n}$, $V_r \gg V_{bi}$, $n = \frac{1}{m+2}$ ⇒ $\omega_0 = \frac{1}{\sqrt{LC}}$

Optoelectronics diode ($V_A > 0$: solar cell, $V_A < 0$: photodetector)

- $I = I_0(e^{\frac{qV_A}{k_BT}}-1) + I_L$, $I_L = -qA(L_P + L_N + W)G_L$
- only minority carriers within 1 diffusion length of DR will reach DR
- γ generates EHP

p-i-n Diode

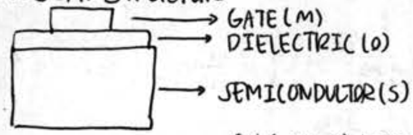
- $W \approx W_i$ ⇒ most carriers generated in DR (not QNR)
- operate near avalanche

LEDs

- compound semiconductors (direct bandgap), operated in forward bias, γ when recombines in QNR
- $\lambda = \frac{1.24}{E_G}$ [μm] ⇐ γ emitted when EHP recombines

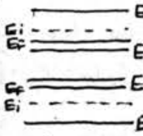
MOS CAPACITOR
*energy band diagrams * C-V characteristics *electrostatics

MOSCAP Structure



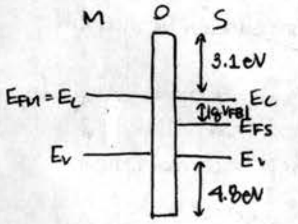
- Gate (Poly-Si) $\phi_M = \{ 4.1\,eV, n^+$ type; $5.2\,eV, p^+$ type $\}$
- Dielectric (SiO_2) : $E_G(SiO_2) = 9\,eV, \varepsilon = 3.9\varepsilon_0$
- Semiconductor (Si) = { p-type, n-channel; n-type, p-channel }

Bulk semiconductor potential
- $-qV_F \equiv E_i(\text{bulk}) - E_F$
  - $V_F = \frac{k_B T}{q} \ln\left(\frac{N_A}{n_i}\right)$ p-type
  - $V_F = -\frac{k_B T}{q} \ln\left(\frac{N_D}{n_i}\right)$ n-type



MOS band diagram rules
- E_F constant (equilibrium)
- Band bending linear in oxide : $\frac{dE}{dx} = 0 \Rightarrow \frac{dE_c}{dx} = k$
- $\vec{E}_{ox} = \frac{\varepsilon_{Si}}{\varepsilon_{ox}} \vec{E}_{Si} \approx 3\vec{E}_{Si}$
- $\phi_B : Si \rightarrow SiO_2 = 3.1\,eV = \chi_{Si} - \chi_{SiO_2}$ (CONDUCTION)
- $\phi_B : Si \rightarrow SiO_2 = 4.8\,eV$ (VALENCE)
- $qV_G = E_{FS} - E_{FM}$

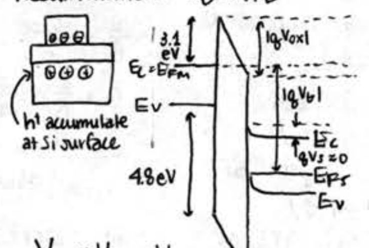
Flat-band condition
- $V_A = V_{FB}$ such that ∄ band-bending
- $qV_{FB} = \phi_M - \phi_S = (\chi_{Si} + E_G)$ $\left[\chi_{Si} + \frac{E_G}{2} + (E_i - E_F)_S\right]$



- $V_G = V_{FB} + V_{ox} + V_{Si}$ ← $= E_i(\text{bulk}) - E_i(\text{surface})$
  - flat band voltage (V_{FB}); band-bending amount ($V_{ox} + V_{Si}$)

C-V characteristics (cont.)
- $N_A / N_D \uparrow$
  - $V_{FB} \downarrow : \phi_S \uparrow \Rightarrow \phi_M - \phi_S \downarrow$
  - $V_T \uparrow : \phi_F \downarrow \Rightarrow V_T \uparrow$
  - $C_{min} \uparrow : W_T \downarrow \Rightarrow C_{dep} \uparrow \Rightarrow C_{min} \uparrow$
- $x_0 \downarrow$:
  - V_{FB} : same
  - $V_T \downarrow : C_{ox} \uparrow$
  - $C_{min} \downarrow : \frac{C_{dep}}{C_{ox}} \downarrow$ since $C_{ox} \uparrow$

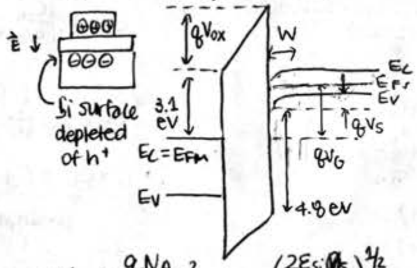
p-type Si : MOS Regions
- Accumulation : $V_G < V_{FB}$



$V_G \approx V_{FB} + V_{ox}$

$Q_{acc} = -C_{ox}(V_G - V_{FB})$ [C/cm²]

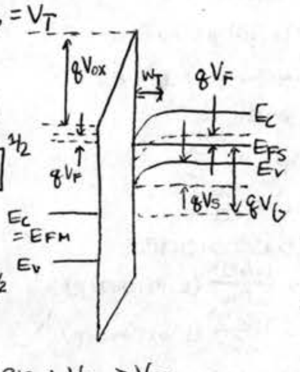
- Depletion : $V_{FB} < V_G < V_T$



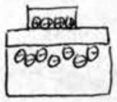
- $\phi_S = \frac{qN_A}{2\varepsilon_{Si}} W^2 \Rightarrow W = \left(\frac{2\varepsilon_{Si}\phi_S}{qN_A}\right)^{1/2}$
- $Q_{dep} = -qN_A W = -\sqrt{2qN_A\varepsilon_{Si}\phi_S}$
- $V_G = V_{FB} + V_{ox} + V_S$, where $V_{ox} = \frac{\sqrt{2qN_A\varepsilon_{Si}\phi_S}}{C_{ox}}$

$qV_S = \phi_S = \frac{qN_A\varepsilon_{Si}}{2C_{ox}^2}\left[\left(1 + \frac{2C_{ox}^2(V_G - V_{FB})}{qN_A\varepsilon_{Si}}\right)^{1/2} - 1\right]^2$

- Threshold : $V_G = V_T$
  - $\phi_S = 2\phi_F = 2\frac{k_B T}{q}\ln\left(\frac{N_A}{n_i}\right)$
  - $W_T = \left[\frac{2\varepsilon_{Si}(2\phi_F)}{qN_A}\right]^{1/2}$
  - $V_T = V_{FB} + 2V_F + \frac{(2qN_A\varepsilon_{Si} \cdot 2V_F)^{1/2}}{C_{ox}}$

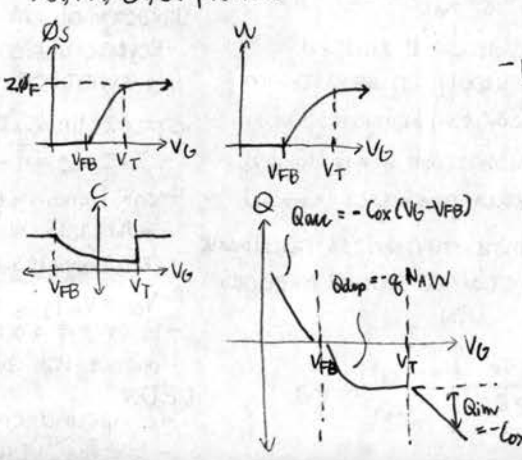


- Strong inversion : $V_G > V_T$



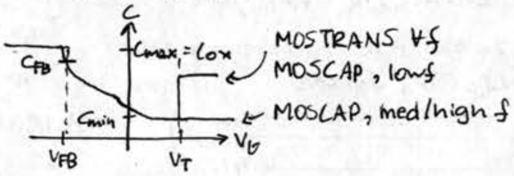
  - $V_S \approx 2V_F$
  - $W_T = \left[\frac{2\varepsilon_{Si}(2\phi_F)}{qN_A}\right]^{1/2}$
  - $Q_{inv} = -C_{ox}(V_G - V_T)$ [C/cm²]

- ϕ_S, W, C, Q profiles

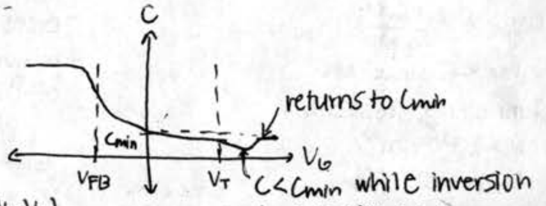


Measuring MOSCAP
- Scan V_G slowly (~0.1V/s) ⇒ Q incrementally added to/subtracted from gate & sub
- $I_{AC} = C\frac{dV_{AC}}{dt}$
- $C = \left|\frac{dQ_G}{dV_G}\right| = \left|\frac{dQ_S}{dV_G}\right|$

C-V characteristics : p-type
- Flat band
  - ΔQ occurs at depth L_D in substrate
  - $L_D = \left(\frac{\varepsilon_{Si} k_B T}{q^2 N_A}\right)^{1/2}$ (Debye length)
  - $C_D = \frac{\varepsilon_{Si}}{L_D}$
  - $\frac{1}{C_{FB}} = \frac{1}{C_{ox}} + \frac{1}{C_D}$
- Depletion
  - ΔQ occurs at depth W in substrate
  - $C = \left|\frac{dQ_{dep}}{dV_G}\right| = \left[\frac{1}{C_{ox}^2} + \frac{2(V_G - V_{FB})}{qN_A\varepsilon_{Si}}\right]^{-1/2}$
  - $\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_{dep}}$
- Inversion
  - CASE 1 : Q_{inv} can be supplied/removed fast enough in response to ΔV_G
    - ↳ ΔQ at SURFACE of substrate
    - ↳ $\tau_{inv,Q} = \frac{2N_A\tau_0}{n_i}$ (time needed to build Q_{inv} via thermal generation)
    - ↳ $C = \left|\frac{dQ_{inv}}{dV_G}\right| = C_{ox}$
  - CASE 2 : Q_{inv} can't be changed fast enough
    - ↳ ΔQ at depth W_T in substrate
    - ↳ $\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_{dep}} = \frac{1}{C_{ox}} + \frac{W_T}{\varepsilon_{Si}}$ $= \frac{1}{C_{ox}} + \left[\frac{2(2\phi_F)}{qN_A\varepsilon_{Si}}\right]^{1/2} = \frac{1}{C_{min}}$
- MOSCAP vs. MOSTRANS



- Quasi-static C-V measurement
  - good for x_0 > 5 nm
  - ramp $V_G \leq 0.1$ V/s while measuring I_G
  - $I_G = C\frac{dV_G}{dt}$
- Deep depletion
  - scanning V_G too quickly ⇒ Q_{inv} can't respond quickly enough
  - ΔQ_S needs to be supplied by ΔQ_{dep}
  - $V_G \uparrow \Rightarrow W \uparrow \Rightarrow C \downarrow$



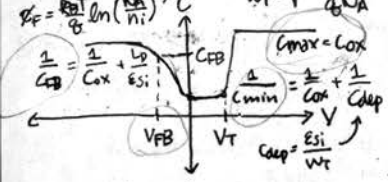
MOS CAPACITOR (cont.) *oxide charges * V_T adjustment * poly-Si gate depletion * diode non-ideal

Effects of oxide charges

- $\Delta V_T = -\frac{1}{\varepsilon_{SiO_2}} \int_0^{x_0} x \rho(x)\, dx$ ← @ metal/oxide interface
  - ↑ gate voltage required to reach threshold
- μ_{eff} of carriers affected due to Coulombic scattering
- Q_F : fixed oxide charge
  $V_{FB} = \phi_{MS} - \frac{Q_F}{C_{ox}}$; V_{FB}, ϕ_{m} vs T_{ox}: $m = \frac{-Q_F}{\varepsilon_{SiO_2}}$

Parameter extraction from C-V

- KNOWN: ε_{SiO_2}, ϕ_M
  $\phi_F = \frac{k_B T}{q} \ln\left(\frac{N_A}{n_i}\right)$; $W_T = \sqrt{\frac{2\varepsilon_{Si}(2\phi_F)}{q N_A}}$



- Q_M : mobile oxide charge
  $\Delta V_{FB} = \phi_{MS} - \frac{Q_F}{C_{ox}} - \frac{1}{\varepsilon_{SiO_2}} \int_0^{x_0} x \rho(x)\, dx - \frac{Q_{it}(\phi_S)}{C_{ox}}$
  $Q_M = C_{ox} \Delta V_{FB}$

Effects of oxide charges (cont.)

- Q_{it} : interface trap charge
  - ↳ traps ~ donor-like: + when empty, neutral when filled
  - ↳ $Q_{it} > 0$ ⇒ shift C-V to left : $V_G \downarrow$ ⇒ $Q_{it} \uparrow$ ⇒ C-V ←
  - ↳ $\Delta V_G = -\frac{Q_{it}(\phi_S)}{C_{ox}}$, degrade μ_{eff}

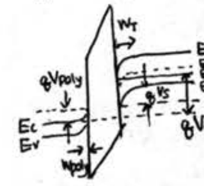
V_T adjustment in ICs via ion implantation in near-surface region of semiconductor

- ↳ $\Delta V_T = -\frac{q N_I}{C_{ox}}$, $N_I = \begin{cases} > 0, \text{ donor} \\ < 0, \text{ acceptor} \end{cases}$
- ↳ when MOS in depletion/inversion, N_I changes (add/subtract) Q_{dep} at O-S interface
- ↳ $V_{T,eff} = V_{FB} + 2\phi_F + \frac{|Q_{dep}|}{C_{ox}} - \frac{q N_I}{C_{ox}}$

Poly-Si Gate ⇒ ~ semiconductor, not metal

↳ Then band diagram changes

Ex: NMOS, inversion



- $Q_{inv} = C_{ox}(V_G - V_{poly} - V_T)$
- $W_{poly} = \sqrt{\frac{2\varepsilon_{Si} V_{poly}}{q N_{poly}}}$
- To minimise gate effect:
  - ↳ increase N_{poly}
  - ↳ use p+ poly ⇒ accumulation ⇒ $\Delta E_G \approx 0$
  - ↳ use metal gate

Poly-Si Gate (cont.)

↳ Gate depletion effect

- $x_0 \to x_0 + \frac{1}{3} W_{poly}$; $\frac{\varepsilon_{r,Si}}{\varepsilon_{r,dielectric}} = \frac{12}{4}$
- ⇒ $C = \left(\frac{1}{C_{ox}} + \frac{1}{C_{poly}}\right)^{-1} = \frac{\varepsilon_{SiO_2}}{x_0 + \frac{1}{3} W_{poly}}$
- $Q_{inv} = (V_G - V_T) \cdot \frac{\varepsilon_{SiO_2}}{x_0 + \frac{1}{3} W_{poly}}$

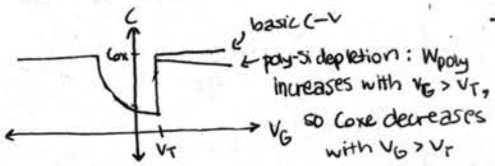
↳ T_{inv} inversion layer thickness = average location of inversion layer below Si/SiO$_2$ interface

↳ T_{oxe} effective oxide thickness

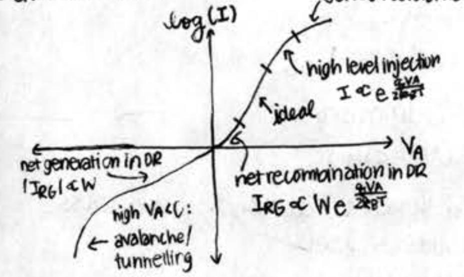
- $T_{oxe} = x_0 + \frac{1}{3} W_{poly} + \frac{1}{3} T_{inv}$
- $T_{inv}(h^+) > T_{inv}(e^-)$ due to $m_h^* > m_e^*$

↳ C_{oxe} effective oxide capacitance

- $Q_{inv} = \int_{V_T}^{V_G} C_{oxe}(V - V_T)\, dV$



Non-ideal diode

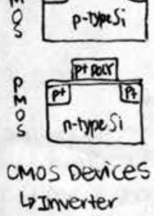


ODE Solutions

- $\frac{\partial^2 y}{\partial x^2} = 0$: $y(x) = Ax^2 + Bx + C$
- $\frac{\partial^2 y}{\partial x^2} = A$: $y(x) = Ax^3 + Bx^2 + Cx$
- $\frac{\partial^2 y}{\partial x^2} = C y(x)$: $y(x) = A_1 e^{\sqrt{C}x} + A_2 e^{-\sqrt{C}x}$
- $C_1 = C_2 y(x) + C_3 \frac{\partial y}{\partial x}$: $y(x) = A + Be^{-\frac{x}{d}}$

MOSFET *structure & operation *field effect mobility *body bias effect *long channel I-V

NMOS vs. PMOS

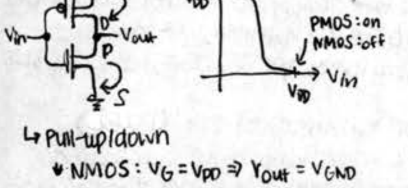


NMOS:
- ON: $V_G > V_T$ to form n-type channel at surface
- ENHANCEMENT: $V_T > 0$
- DEPLETION: $V_T < 0$

PMOS:
- ON: $V_G < V_T$ to form p-type channel at surface
- ENHANCEMENT: $V_T < 0$
- DEPLETION: $V_T > 0$

CMOS Devices

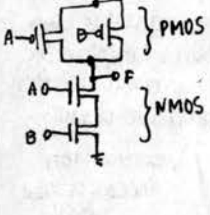
↳ Inverter



↳ Pull-up/down

- NMOS : $V_G = V_{DD}$ ⇒ $V_{out} = V_{GND}$
- PMOS : $V_G = V_{DD}$ ⇒ $V_{out} = V_{DD}$

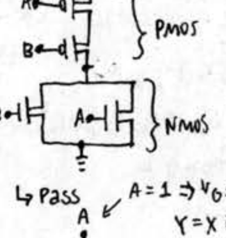
↳ NAND



A	B	F	
0	0	1	A/B low ⇒ PMOS turn on to pull up F
0	1	1	
1	0	1	
1	1	0	→ A & B high ⇒ NMOS turn on to pull down F

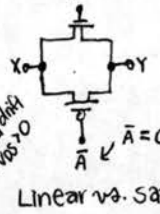
CMOS Devices (cont.)

↳ NOR



A	B	F	
0	0	1	→ A & B low ⇒ PMOS on to pull up
0	1	0	A/B high ⇒ NMOS to pull down
1	0	0	
1	1	0	

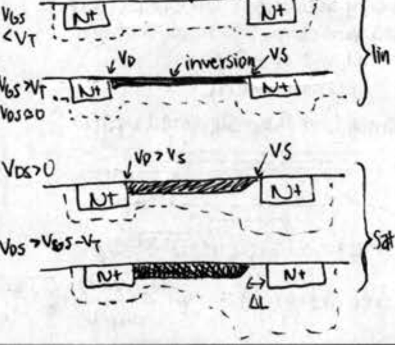
↳ Pass



- $A = 1$ ⇒ $V_{GS} = V_{DD}$ if $X = V_{GND}$
- $Y = X$ if A
- $\bar{A} = 0$ ⇒ $A = 1$ ⇒ $V_{GS} = V_{GND}$

Linear vs. saturation (NMOS)

e^- flow via drift when $V_{DS} > 0$



Body bias

- MOS inversion layer in contact with doped region of same type ⇒ can apply V_B to this pn junction
- creates 2 Fermi levels E_{Fn}, E_{Fp} ⇒ $|E_{Fn} - E_{Fp}| = qV_{BC}$
- ΔV_T ⇒ $V_T(y) = V_{FB} + V_{CB}(y) + 2\phi_F + \frac{\sqrt{2 q N_A \varepsilon_{Si}(2\phi_F + V_{CB}(y))}}{C_{ox}}$

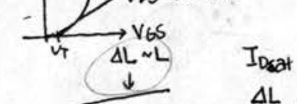
(N)MOSFET I-V

- $I_{DS} = \begin{cases} I_{Dlin} = W Q_{inv} v = W Q_{inv} \mu_{eff} E = W Q_{inv} \mu_{eff} \left(\frac{V_{DS}}{L}\right) \\ I_{Dsat} = \frac{W}{2mL} C_{oxe} \mu_{eff} (V_{GS} - V_T)^2 [1 + \lambda(V_{DS} - V_{Dsat})] \end{cases}$
- *linear*: $V_{DS} < \frac{V_{GS} - V_T}{m}$, *saturated*: $V_{DS} \geq \frac{V_{GS} - V_T}{m}$
- $Q_{inv} = C_{oxe}(V_{GS} - V_T - \frac{1}{2} m V_{DS})$
- $m = 1 + \frac{C_{dep,min}}{C_{oxe}} = 1 + 3\frac{T_{oxe}}{W_T}$

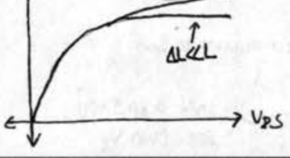
Body effect

- $V_T = V_{T_0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$
- $\gamma = \frac{\sqrt{2 q N_A \varepsilon_{Si}}}{C_{ox}}$

Determine V_T



- choose $V_{DS} \ll$, then linearly extrapolate



$I_{Dsat} = I_{Dsat0}[1 + \lambda(V_{DS} - V_{Dsat})]$

$\frac{\Delta L}{L} \equiv \lambda(V_{DS} - V_{Dsat})$

[channel length modulation]

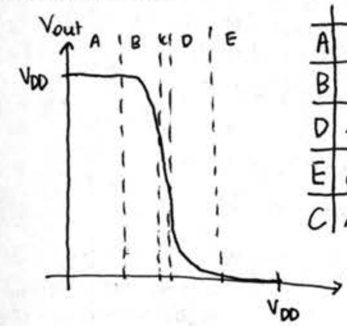
MOSFET (cont.)

* PMOSFET I-V * subthreshold current * velocity saturation * S/D structure
* CMOS inverter * MOSFET scaling * short channel effects

PMOSFET I-V (Long Channel)

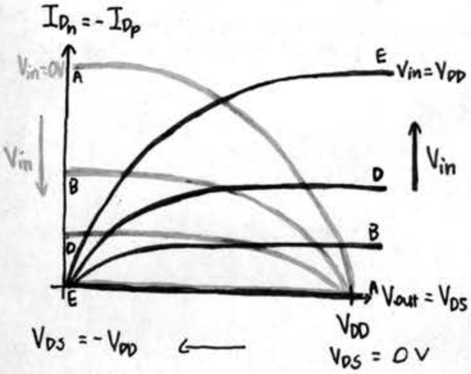
- $I_{DS} = \begin{cases} I_{Dlin} = -\frac{W}{L} C_{oxe}\mu_{p,eff}(V_{GS}-V_{Tp}-\frac{1}{2}mV_{DS})V_{DS} \\ I_{Dsat} = -\frac{W}{2mL} C_{oxe}\mu_{p,eff}(V_{GS}-V_{Tp})^2 \end{cases}$
- $m = 1 + 3\frac{T_{oxe}}{W_T}$

CMOS Inverter

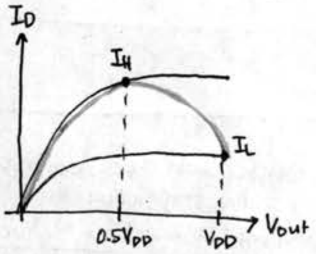


	P	N	Vin
A	lin	off	$\le V_{Tn}$
B	lin	sat	$V_{Tn} < V_{in} < \frac{V_{DD}}{2}$
D	sat	lin	$\frac{V_{DD}}{2} < V_{in} < V_{DD} - \lvert V_{Tp}\rvert$
E	off	lin	$> V_{DD} - \lvert V_{Tp}\rvert$
C	sat	sat	

$I_{Dn} = -I_{Dp}$



- $I_{eff} = \frac{I_H + I_L}{2}$



- τ_d propagation delay
  - ↳ CMOS inverter chain

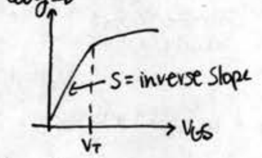
$2\tau_d = (t_{pHL} + t_{pLH}) \propto \frac{CV_{DD}}{I_{eff}}$

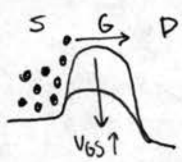
Subthreshold Current

- $|V_G| < |V_T| \Rightarrow I_S$ limited by carrier diffusion into channel
- $\Delta V_C = \frac{C_{oxe}}{C_{oxe} + C_{dep}} \Delta V_G = \frac{1}{m}\Delta V_G$ ↳ linearly
- increased potential barrier to diffusion $\Rightarrow I_{DS} \propto e^{\frac{qV_G}{mk_BT}}$
- S subthreshold swing

$$S \equiv \left[\frac{d(\log_{10} I_{DS})}{dV_{GS}}\right]^{-1} = \frac{k_BT}{q}\ln(10)\left(1 + \frac{C_{dep,min}}{C_{oxe}}\right)$$

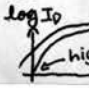
($\frac{C_{dep,min}}{C_{oxe}} = 3\frac{T_{oxe}}{W_T}$)





V_T Trade-off

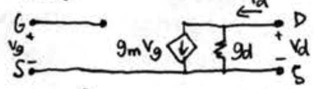
- $I_{Dsat} \propto (V_{DD} - V_T)^{\eta}$ ⇒ minimise V_T to maximise I_{on}
- log ID



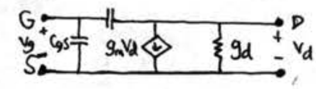
↳ high V_T needed for low I_{off} → can't aggressively scale down V_T

MOSFET Small Signal

- low f



- high f



- $i_d = g_d V_d + g_m V_g$
- $g_d = \lambda I_{Dsat_0}$
- $g_m = \frac{W}{mL}\mu_{eff} C_{oxe}(V_G - V_T)$

MOSFET Cutoff Frequency

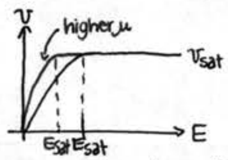
- $\beta = 1: \frac{j\omega C_G V_G}{g_m V_G} = 1$
- $\Rightarrow f_T \approx \frac{g_m}{2\pi C_G} = \frac{\mu_{eff}}{2\pi m L^2}(V_G - V_T)$

MOSFET Scaling

- constant field: scale every dimension & V_{DD} by K so that $\vec{E} = \frac{V}{d} \Rightarrow$ same
- generalised: $\vec{E}$ scaled by $\alpha > 1$, N_{body} by α to suppress short channel effects

Velocity Saturation

- $v = \begin{cases} \frac{\mu E}{1 + \frac{E}{E_{sat}}}, & E < E_{sat} \\ v_{sat} = \frac{\mu E_{sat}}{2}, & E \ge E_{sat} \end{cases}$
- $v_{sat} = \begin{cases} 8\times10^6\ cm/s,\ e^-\ in\ Si \\ 6\times10^6\ cm/s,\ h^+\ in\ Si \end{cases}$



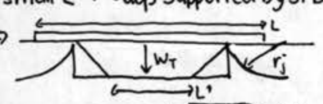
- $I_{DS} = \begin{cases} \frac{\text{long-channel } I_{DSlin}}{1 + \frac{V_{DS}}{E_{sat}L}}, & lin+LC \\ \frac{\text{long-channel } I_{Dsat}}{1 + \frac{V_{GS}-V_T}{mE_{sat}L}}, & sat\ LC \\ Wv_{sat}C_{oxe}(V_{GS}-V_T), & sat\ SC \end{cases}$

  $v_{sat} = \frac{\mu E_{sat}}{2}$ ⇓ $I_{Dsat} \propto V_{GS} - V_T$, not $()^2$; $I_{Dsat} \not\propto L$

- $\frac{1}{V_{Dsat}} = \frac{m}{V_{GS} - V_T} + \frac{1}{E_{sat}L}$ — (pinchoff V)$^{-1}$, (velocity saturation)$^{-1}$

Short channel effect

- $I_{Dsat} \propto (V_{GS} - V_T)$, $\not\propto L$
- $V_{Dsat}(SC) < V_{Dsat}(LC)$
- velocity overshoot: short L (< MFP) causes some carriers to travel through channel w/o collision
- $|V_T|$ decreases with L
  - ↳ small L ⇒ Q_{dep} supported by S/D ↑

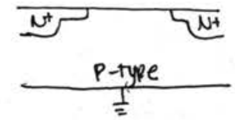


$L' = L - 2r_j\left(\sqrt{1 + \frac{2W_T}{r_j}} - 1\right)$

↳ $\Delta V_T = |V_T| - |V_{T,LC}| = -q\frac{N_{A/D}W_T r_j}{C_{oxe}L}\left(\sqrt{1 + \frac{2W_T}{r_j}} - 1\right)$

S/D Structure

- $R_{S/D} \propto \frac{\rho}{Wr_j}$
  - ↳ want small r_j but it increases R_{paras}
  - ↳ solution: shallow S/D extension regions to reduce r_j but with smaller R_{paras}



Lightly doped drain structure (LDDS)

- Lateral $\vec{E}$ peaks at D region
  - ↳ too high $\vec{E}$ ⇒ damage to oxide interface & bulk
  - ⇒ ∃ substrate current due to impact ionisation
- LDD lowers $\vec{E}$ but increases R_{paras}

Parasitic S/D Resistance

- $I_{Dsat}' = \frac{I_{Dsat0}}{1 + \frac{I_{Dsat0}R_S}{V_{GS} - V_T}}$ [SC]
- $V_{Dsat}' = V_{Dsat_0} + I_{Dsat}'(R_S + R_D)$
- R_S reduces V_{GS}, V_{DS}
- R_D reduces V_{DS}

ON/OFF Summary

- OFF ($V_{GS} < V_T$) ← PMOS
  - ↳ I_{DS} limited by carrier diffusion across S
  - ↳ issues: S, DIBL
- ON ($V_{GS} > V_T$)
  - ↳ I_{DS} limited by carrier drift across channel
  - ↳ issues
    - *punchthrough at high V_D
    - *parasitic R reduces I_{drive}

CMOS Technology

- performance boosters
  - ↳ strained channel regions ⇒ μ_{eff} ↑
  - ↳ high-k gate dielectric, metal gate electrode ⇒ C_{oxe} ↑
  - ↳ parallelism (run multiple cores at different V_{DD}) to get around power dissipation limits
  - ↳ reduce V_{DD} via gate control (capacitively couple gate & channel) ⇒ lower V_{DD} necessary for target I_{on}/I_{off}, reduce SCE & DIBL

Short-channel effect (cont.)

- L↓ ⇒ S/D coupling ⇒ V_D can affect potential barrier to carrier diffusion at S
- $DIBL \equiv \frac{|V_{Tlin}| - |V_{Tsat}|}{V_{DD} - V_{Dlin}}$
- punchthrough
  - ↳ large V_D ⇒ drain junction DR can merge with source junction DR ⇒ new pathway for current conduction
  - ↳ mitigate using retrograde doping