EE 147/247A: MEMS
Final Exam Study Guide

Alison Kim

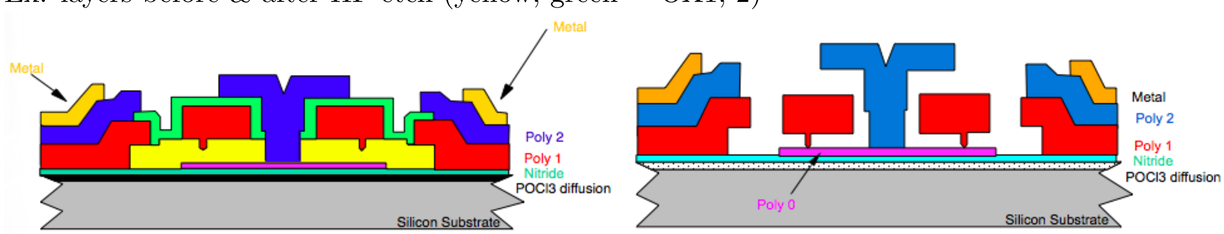
Layouts & Cross Sections

Drawing Conventions

- Draw conductors (POLY0/1/2, METAL)
- Holes in dielectrics, insulators (ANCHOR1/2, P1P2VIA, DMP)
- Etch cuts through sacrificial layers (Ox1/2)

POLYMUMPS

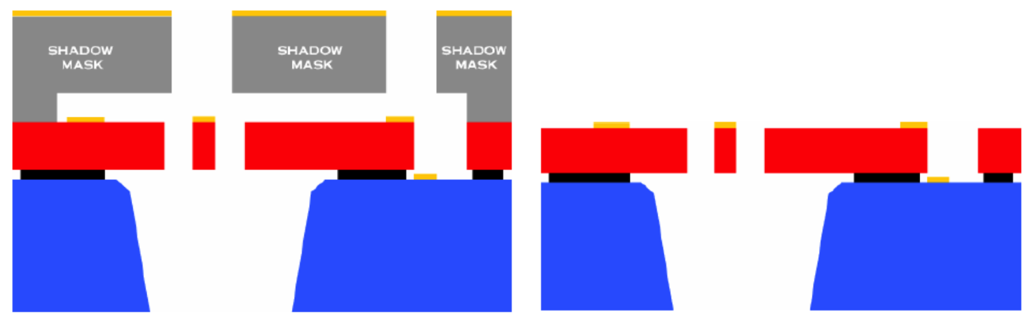
- LEVEL ≡ lithographic level used to pattern a physical layer, may or may not itself be a physical layer (e.g., VIA is not a physical layer)
- Layer ≡ physical layer of material that is deposited
- LEVELS & Layers
  - Nitride: isolates substrate from electrical surface layers
  - POLY0: defines poly-Si 0 features
  - Ox1: first sacrificial oxide layer that provides gap between POLY1 & substrate/nitride
  - ANCHOR1: opens contact between sub, POLY1
  - DIMPLE: generates bumps in under-surface of POLY1 to minimise stiction
  - POLY1: defines poly-Si 1 features
  - Ox2: second sacrificial oxide layer that provides gap between POLY1, 2
  - P1P2VIA: opens contact between POLY1, 2
  - ANCHOR2: opens contact between substrate/nitride, POLY2
  - POLY2: defines poly-Si 2 features
  - METAL: defines location of metal features
- Ex: layers before & after HF etch (yellow, green = OX1, 2)



Layouts & Cross Sections (cont.)

SOIMUMPS

- Silicon-on-insulator (SOI) wafer used as starting substrate
  - Si thickness: $10\mu m$ or $25\mu m$
  - Oxide thickness: $1\mu m$ (for $10\mu m$ Si) or $2\mu m$ (for $25\mu m$ Si)
  - Substrate thickness: $400\mu m$
- Silicon layer pattern & etch to oxide layer: can serve as mechanical structure, resistor structure, or electrical routing
- Substrate patterned & etched from "bottom" side to oxide layer: allows for through-hole structures
- Shadow-masked metal process provides metal features (e.g., bond pads, electrical routing)
  - E.g., layers before & after shadow mask bonding + metal deposition



Accelerometers

- $V \propto \frac{dC}{dx}$
- Total capacitance $C(x)$
  - $C_{finger} = N\epsilon_0 \frac{A}{g}$ (N = # fingers, A = overlapping area, g = distance between fingers)
  - $x = \frac{ma}{k_{sp}}$ [displacement]
  - $m = \rho V$ [mass of plate]
  - $C(x) = C_0(1 + \frac{x}{g})$ [total capacitance]

Parallel Plate Structures

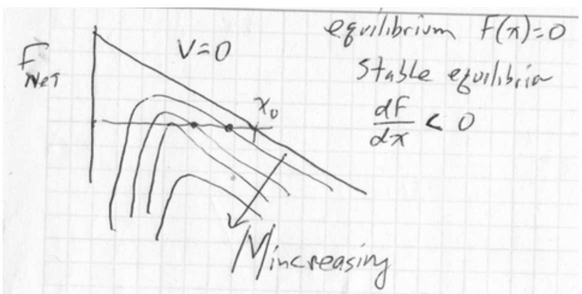
Electrostatics

- $C = \epsilon_0 \frac{A}{g} = \epsilon_0 x \frac{t}{d}$ [capacitance]
- $E_{cap} = \frac{V}{g}$ [electric field between plates]
- $U_{cap} = \frac{1}{2}CV^2$ [potential energy storage]
- $F_{cap} = -\frac{dU}{dq} = \frac{dU_{cap}}{dx} = \frac{d}{dx}(\frac{1}{2}CV^2)$ [electrostatic force]
- E_{fringe} adds $\frac{1}{2}g$ of width to each side of plate

Parallel Plate Structures (cont.)

Gap Closer

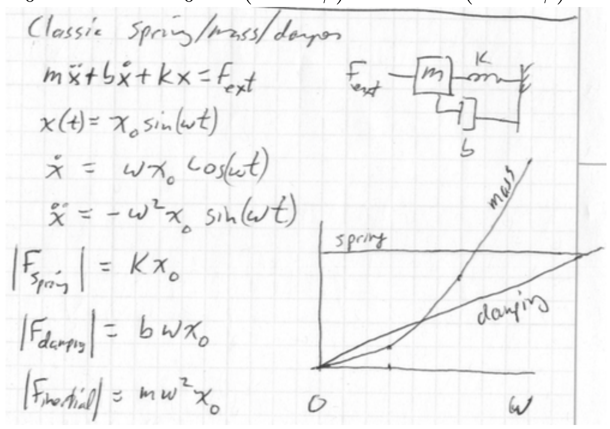
- $F_{total} = F_{spring} + F_{elec} = k(x_0 - x) - \frac{1}{2}\epsilon_0 V^2 \frac{A}{x^2}$
- $V_{pi} = \sqrt{\frac{kx_{pi}^3}{\epsilon_0 A}} = \sqrt{\frac{8}{27}\frac{kx_0^3}{\epsilon_0 A}}$ [pull-in voltage]
- $x_{pi} = \frac{2}{3}x_0$ [pull-in displacement]
- $\Delta x = \frac{1}{3}x_0$ [maximum stable displacement]
- V_{po} : want $F_{elec} = F_{spring}$; use F_{spring}, F_{spring} relations before & after pull-in to determine
- $\frac{1}{2}\epsilon_0 V^2 \frac{A}{(g_0 - g_{stop})^2} < kg_{stop}$ [bistability condition]



TNH Comb Drive Resonator

- $k = 8k_0 = 8\frac{Ea^3b}{4L^3}$
- $\omega_n = \sqrt{\frac{k}{m}} \Rightarrow \omega_n^2 = \frac{Ea^3b}{4L^3m}$
- $x_{DC} = \frac{F_0}{k}$ [DC deflection]
- $x_{\omega n} = \frac{F_0}{\omega_n b}$ [resonant deflection]
- $Q = \frac{d}{\mu A_{plate}}\sqrt{m_{plate}k} = \frac{x_0(\omega_n)}{x_0(\omega<\omega_n)} = \frac{x_{\omega_n}}{x_{DC}} = \frac{k}{\omega_n b}$
- $b = \frac{\mu A_{plate}}{g}$
- $\omega^{\frac{2}{3}} \propto \alpha + \delta\alpha$ [line width offset]
- $F_{elec} = 2N_f(\frac{1}{2}\epsilon_0 V^2)\frac{t}{g}$
- $F \propto V^2 = (V_{AC}\sin\omega t + V_{DC})^2$
  - $V(t)^2 = DC + AC(\omega) + AC(2\omega)$
    - DC: $\frac{1}{2}V_{AC}^2 + V_{DC}^2$
    - $\omega : +2V_{AC}V_{DC}\sin\omega t$
    - $2\omega : -\frac{1}{2}V_{AC}^2\cos 2\omega t$
  - $F(t) = F_0\sin\omega t \Rightarrow x(t) = x_0\sin(\omega t + \phi)$
    - $\omega \ll \omega_n : x_0 = \frac{F_0}{k}, \phi = 0$ [spring]
    - $\omega = \omega_n : x_0 = \frac{F_0}{b\omega_n}, \phi = -\frac{\pi}{2}$ [damping]
    - $\omega \gg \omega_n : x_0 = \frac{F_0}{m\omega^2}, \phi = -\pi$ [inertial]

- $F_0 \sin \omega t = kx_0 \sin(\omega t + \phi) + b\omega \cos(\omega t + \phi) - m\omega^2 \sin(\omega t + \phi)$



- $I(t) = \frac{d(CV)}{dt} = 2N_f V_{DC} \epsilon_0 a \dot{x}(t)$

Euler-Bernoulli Beam Theory

- $F = kx$
- $k_{xx} = \frac{EA}{L}$
- $k_{yy} = k_{zz} = \frac{EI}{4L^3} = \frac{Ea^3 b}{4L^3}$
- $k_{z\theta} = \frac{F_0}{\theta} = \frac{Ea^3 b}{6L^2}$
- $\epsilon(z) = \frac{\Delta L}{L} = \frac{z}{\rho}$
- $\kappa = \frac{1}{\rho(x)} \approx \frac{d^2 y}{dx^2}$ (curvature, small deflection)
- $M(x) = M_0 + F(L - x) = \frac{EI}{\rho(x)}$
  - $\frac{1}{\rho(x)} \approx \frac{d^2 y}{dx^2} = \frac{M_0 + F(L-x)}{EI}$
  - $EI\frac{dy}{dx} = M_0 x + FLx - \frac{1}{2}Fx^2$
  - $EIy(x) = \frac{1}{2}(M_0 + FL)x^2 - \frac{1}{6}Fx^3$
- $\epsilon_{max}(x, z) = \epsilon(0, \pm\frac{1}{2}a)$

Materials, Stress & Strain

Young's moduli for various materials

Material	Young's modulus (GPa)
Poly-Si	150*
Single-crystal Si	130-190
Steel	200
Aluminium	70
SiO_2	70
Diamond	1200
Polymers	≈ 1
Wood, bone	≈ 100

*Roughly. See **Crystal Planes** section.

Materials, Stress & Strain (cont.)

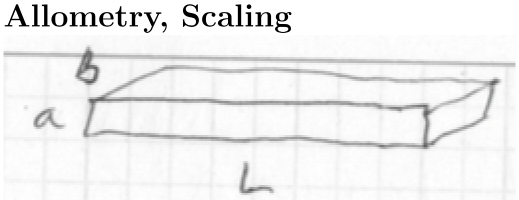
Material Properties

- $\sigma = \frac{F_x}{A}[\frac{N}{m^2}]$ (normal stress)
- $\tau = \frac{F_y}{A}[\frac{N}{m^2}]$ (shear stress)
- $\epsilon = \frac{\Delta L_x}{L} = \frac{x}{L}$ (normal strain)
- $\gamma = \frac{\Delta L_y}{L} = \frac{y}{L} \approx \theta_{deflec}$ (shear strain)
- $\nu = -\frac{\epsilon_y}{\epsilon_x}$ (Poisson ratio)
- $R = \rho\frac{L}{A}$ (resistance)
- $\sigma = E\epsilon$ (linear isotropic materials)

Residual Stress, Stress Gradients, Buckling

- $F_{crit} = K\pi^2\frac{EI}{L^2} = \frac{K\pi^2 Ea^3 b}{12L^2}$ (K = 4, thin film)
  - Will be the largest for a fixed-fixed structure (e.g., clamp-clamp beam)
- $\sigma_0 = \frac{F_{crit}}{A}$ (average residual stress, thin film)
- $\rho(x) = \frac{L^2}{2y(x)}$
- $\sigma(z) = \sigma_1\frac{z}{L} = \frac{Ea}{2\rho(L)}\frac{z}{L}$ (stress gradient)
  - $\sigma(z)$: slope of gradient is $\pm \Rightarrow$ contracts/expands
  - σ_0: top of beam in T/C region $\Rightarrow$ curl up/down
- Compressive: $\sigma < 0$
  - Substrate shrinks more than film $\Rightarrow$ film wants to shrink
  - Curls down $\Rightarrow$ buckling
- Tensile: $\sigma > 0$
  - Substrate shrinks less than film $\Rightarrow$ film wants to expand
  - Curls up $\Rightarrow$ wants to rip apart since $\overrightarrow{F}$ outward

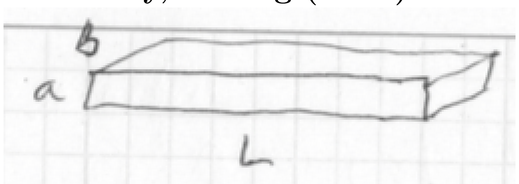
Allometry, Scaling



Scale every dimension of structure by S

Feature	k	m	b	F	ω_n	Q	Δx_{DC}	Δx_{ω_n}
$\propto$	$\frac{hw^3}{L^3}$	V	A	$\frac{h}{g}$	$\sqrt{\frac{k}{m}}$	$\frac{k}{b\omega}$	$\frac{F}{k}$	$\frac{F}{b\omega}$
Scaling Factor	S	S^3	S^2	*same*	S^{-1}	*same*	S^{-1}	S^{-1}

Allometry, Scaling (cont.)



Scale only Z dimension by S

Feature	k, m, F	b	ω_n	Q	Δx_{DC}	Δx_{ω_n}
$\propto$	h	b	$\sqrt{\frac{k}{m}}$	k	$\frac{F}{k}$	F
Scaling Factor	S	*same*	*same*	S	*same*	S

Scaling k, m by S; given $L_{beam} \propto S$, $m_{plate} \propto S^3$

Feature	k_{xx}	k_{yy}	k_{zz}	$k_{\theta z}$	F	M	Δz	$\Delta\theta_z$	dz_{plate}	a_{-x}	a_y
$\propto$	$\frac{A}{L}$	$\frac{hw^3}{L^3}$	$\frac{h^3w}{L^3}$	$\frac{h^3w}{L^2}$	m	mL	$\frac{FL^3}{I}$	$\frac{FL^2}{I}$	$\theta_z L$	$\frac{A}{m}$	$\frac{I}{hmL}$
Scaling Factor	S	S	S	S	S^3	S^4	S^2	S	S^2	S^{-1}	S^{-1}

Sensors: Thermal, Piezoresistive, Strain

- $R(\epsilon) \approx R_0[1 + \epsilon(1 + 2\nu)]$
- $V_A = A(V_+ - V_-)$ (Wheatstone bridge)
  - $V_+ = \frac{1}{2}V_x + \frac{1}{4}G\epsilon V_x$
  - $V_- = \frac{1}{2}V_x$
- $P = \frac{V^2}{R}$
- $\frac{\Delta R}{R} = \alpha \Delta T$ (α = temperature coefficient of resistance) [thermal change]
- $\frac{\Delta R}{R} = G\epsilon$ [piezoresistance]
- $R_{th} = \frac{L}{kab}$
- $\Delta T = R_{th}\dot{Q} = \frac{L}{kab}\dot{Q}$ [thermal conductivity]

Crystal Planes

Crystal Planes

- {100}
  - Lowest interface states with SiO2
  - Best for CMOS
  - Most common choice
- {110}
  - Mechanically weakest direction
  - Wafers like to cleave along this direction

- {111}
  - Stiffest direction
  - Slowest etching in KOH, EDP, and other orientation-dependent etches
  - Used for bipolar circuits
- $a_0 \approx 0.543nm$ [lattice constant of Si at $T = 300K$]

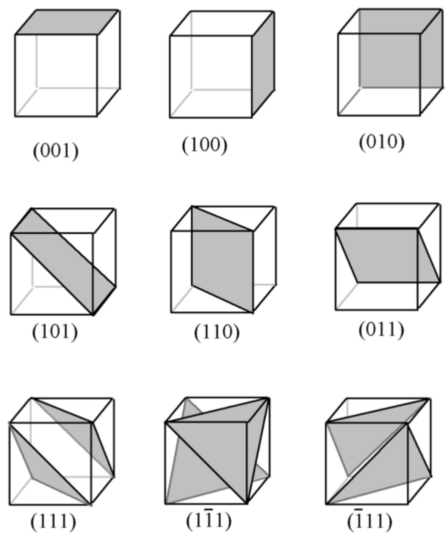


Figure 1: Crystal planes. (Source: http://nano-physics.pbworks.com/f/1242359145/miller%20index-cubic.png)

Orientation Dependence of Young's Modulus

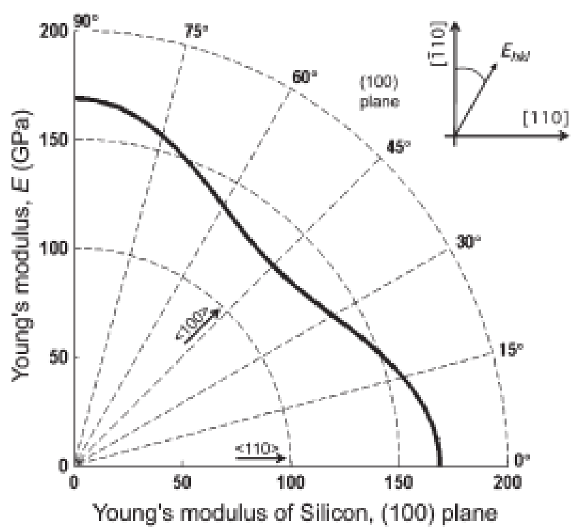


Figure 2: Orientation dependence of Young's modulus of Si. (Source: M. Hopcroft, W. Nix, T. Kenny, "What is the Young's modulus of silicon?", JMEMS V19N2, April 2010.)

MEMS Fabrication

CMOS MEMS

- CMOS: Complementary Metal-Oxide-Semiconductor
  - Obtaining mobile electrons in a semiconductor
    - Illumination: γ absorption
    - Thermal energy
    - Impurities (a.k.a. dopants)
    - Apply $\vec{E}$-field to generate carriers
  - Types
    - Thin film dielectric metalisation structures
    - SOI structures wafer-bonded to CMOS
- Fedder process
  - Post-processing on CMOS to achieve final MEMS product
  - Structural materials: metal, dielectrics
  - Sacrificial materials: dielectrics between structures, silicon beneath structures
  - Etchants: RIE (dielectrics), dry SF_6:O_2 plasma (silicon)
- Invensense/Nasiri process
  - Seal MEMS device on CMOS using eutectic Ge onto Al on CMOS
- Cost: $\approx$ \$0.05/mm^2!

Processing

- PR spinning, patterning

Deposition

Mechanism	**Substrate Potential**	**Gases**	**Conformal / Directional?**	**Chemical / Kinetic?**	**Compatibility**
Evaporation	N/A	N/A	Directional	Chemical	Al, Cu, Au
Sputtering	GND	Noble (inert) gases	Conformal	Kinetic	?
LPCVD	Wafer at GND	SiH_4	Highly conformal	Chemical	SiH_4 can be amorphous ($< 580°$), polycrystalline ($580 - 630°C$), single crystal ($> 1000°C$)
LTO	Wafer at GND	$SiH_4 + O_2 \rightarrow SiO_2$, $SiH_4 + O_2 + PH_3 \rightarrow$ PSG	Poorly conformal	Chemical	$< 450°C$, Al
HTO	Wafer at GND	$SiH_2Cl_2 + N_2O \rightarrow SiO_2$, $SiH_2Cl_2 + NH_3 \rightarrow Si_3N_4$	Highly conformal	Chemical	$850°C$, Al

Etching

Mechanism	Substrate Potential	Gases	Chemical / Kinetic?	Isotropic / Anistropic?	Selectivity
XeF_2	N/A	Noble (inert) gases	Chemical	Isotropic	Selective to PR, SiO_2, Al
Hydrofluoric-Nitric-Acetic Acids (e.g., HF, HNOs, CH_3COOH)	N/A	N/A	Chemical	Isotropic	Somewhat selective
KOH	N/A	N/A	Chemical	Anisotropic	Selective to Si (fastest on {100}, slowest on {111})
Sputtering	$V_{DC} \neq 0$	Noble (inert) gases	Kinetic	Anisotropic	Non-selective
Plasma	GND	Noble (inert) gases, halogens	Chemical	Isotropic	Selective
RIE	Floating	Noble (inert) gases, halogens	Chemical, somewhat kinetic	Typically anisotropic	Selective
DRIE	Low power RF bias	SF_6 (etch), C_4F_8 (passivation deposition)	Chemical, somewhat kinetic	Isotropic (Si etch, deposition), anisotropic (Teflon etch)	Selective

Stringers

- Form when a conformal film that covers features is etched directionally (e.g., RIE, plasma)
- Remove with sonication

Stiction

Method	Stage	Reason
CO_2 critical point drying	Release	Avoids the meniscus problem, which is conducive to stiction
Dimples	Release, operation	Increase strain energy of structure, which reduces L_{eff} (i.e., processed length) and device-substrate overlapping surface area
Dry etch (XeF_2, vapour HF, etc.)	Release	Gas phase etch $\Rightarrow$ no liquid
Self-assembled monolayers (OTS, DDMS, etc.)	Release, operation	Permanently make surfaces hydrophobic, which reduces stiction

Noise

- Sources
  - Environmental (e.g., $60Hz$ noise)
  - Shot noise ($\bar{I}^2_{noise} = 2qI\Delta f$)
  - Thermal noise (e.g., Brownian motion, Johnson noise, white noise)
- Equipartition
  - Average thermal energy of $\frac{1}{2}k_BT$ for every quadratic energy storage element
  - Capacitor ($\frac{1}{2}CV^2$), spring ($\frac{1}{2}kx^2$), kinetic ($\frac{1}{2}mv^2$), inductor ($\frac{1}{2}LI^2$)
- Fluctuation & Dissipation
  - Noise power in a dissipative element introduces fluctuation in the system: $\bar{P}_N = 4k_BT\Delta f$
  - $\frac{\bar{P}_N}{\Delta f} = 4k_BT$ [power spectral density]
  - $\bar{V}_N = \sqrt{4k_BTR\Delta f}$ [resistor]
  - $\bar{V}_N = \sqrt{4k_BTb\Delta f}$ [spring-damper]
- $a_{N,eq} = \frac{\bar{V}_{N,out}}{gain}$ [noise-equivalent acceleration]