$$\sum_{k=0}^{K} a_k y(n-k) = \sum_{m=0}^{M} b_m x(n-m) \Leftrightarrow H(\omega) = \frac{\sum_{m=0}^{M} b_m e^{-i\omega m}}{\sum_{k=0}^{K} a_k e^{-i\omega k}}$$

$$\cos(x) = \frac{e^{ix}+e^{-ix}}{2}, \sin(x) = \frac{e^{ix}-e^{-ix}}{2i}$$

$x(n)$ real and odd function $\Rightarrow$ odd imaginary DTFS/CTFS coefficients

$x(n)$ real and even function $\Rightarrow$ real and even DTFS/CTFS coefficients

$$x(n) = X_0\psi_0 + X_1\psi_1 + \cdots + X_{p-1}\psi_{p-1}$$

DTFS:

$$x(n) = \sum_{k=0}^{p-1} X_k \cdot \psi_k(n) = \sum_{k=0}^{p-1} X_k \cdot e^{ik\omega_0 n}$$

$$X_k = \frac{1}{p}\sum_{n=0}^{p-1} x(n) \cdot e^{-ik\omega_0 n}$$

CTFS:

$$x(t) = \int_{<p>} X_k \cdot e^{ik\omega_0 t}\, dt$$

$$X_k = \frac{1}{p}\int_{<p>} x(t) e^{-ik\omega_0 t} dt$$

$$\sum_{k=A}^{B} \alpha^k = S = \frac{\alpha^{B+1}-\alpha^A}{\alpha - 1}$$

$$y(n) = x(n-m) \Leftrightarrow Y_k = e^{-ik\omega_0 m} \cdot X_k$$

$$y(n) = e^{im\omega_0 n} x(n) \Leftrightarrow Y_k = X_{k-m}$$

$$< x, y > = < y, x >^*$$

$$< x_1 + x_2, y > = < x_1, y > + < x_2, y >$$

$$< \alpha x, y > = \alpha \cdot < x, y >$$

$$< x, \alpha y > = \alpha^* \cdot < x, y >$$

$$< f \pm g, f \pm g > = < f, f > + < g, g > \pm < g, f > \pm < f, g >$$

$$g(n) = \alpha^n u(n), |\alpha| < 1 \Leftrightarrow G(\omega) = \frac{1}{1-\alpha e^{-i\omega}}$$

$$(x \star h)(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

$$\text{DTLTI system BIBO } \Leftrightarrow \sum_{n\in\mathbb{Z}} |h(n)| < \infty$$

$$\text{Memoryless} \Leftrightarrow \text{Depends only on } x(t)$$

DTFT:

$$x(n) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} X(\omega) e^{i\omega k}$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} x(k) e^{-i\omega k}$$

CTFT:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt$$

$$x_1(n) \cdot x_2(n) \leftrightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\theta) X_2(\omega - \theta) d\theta$$

CTFT Properties:

$$\hat{x}(t) = x(a \cdot t) \Leftrightarrow \hat{X}(\omega) = \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

$$\hat{x}(t) = \frac{dx}{dt} \Leftrightarrow \hat{X}(\omega) = i\omega X(\omega)$$

$$\hat{x}(t) = t^N \cdot x(t) \Leftrightarrow \hat{X}(\omega) = \frac{i^N d^N X(\omega)}{d\omega^N}$$

$$x(t) \leftrightarrow X(\omega) \Longrightarrow X(t) \leftrightarrow 2\pi x(-\omega)$$

Parseval's

$$< x, y > = \int_{-\infty}^{\infty} x(t) y^*(t) dt \text{ and } < X, Y > = \int_{-\infty}^{\infty} X(\omega) Y^*(\omega) d\omega$$

$$\Rightarrow < x, y > = \frac{1}{2\pi} < X, Y >$$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

Stuff:

$$A\cos(\omega_0 t + \alpha) + B\sin(\omega_0 t + \alpha) = \sqrt{A^2+B^2} \cdot \cos\left(\omega t + \arctan\left(\frac{A\sin(\alpha) - B\cos(\alpha)}{A\cos(\alpha) + B\cos(\alpha)}\right)\right)$$

$$\int_{-\infty}^{\infty} e^{-i\omega t} dt = 2\pi\delta(\omega)$$

$$\sum_{m=a}^{b} \alpha^m = \begin{cases} b - a + 1 & \alpha = 1 \\ \dfrac{\alpha^{b+1}-\alpha^a}{\alpha - 1} & \alpha \neq 1 \end{cases}$$

Common Pairs:

$$x(t) = \operatorname{sgn}(t) \leftrightarrow X(\omega) = \frac{2}{i\omega}$$

$$x(t) = u(t) \leftrightarrow X(\omega) = \frac{1}{i\omega} + \pi\delta(\omega)$$

$$x(t) = \begin{cases} b & |t| < a \\ 0 & \text{elsewhere} \end{cases} \leftrightarrow X(\omega) = 2ab \cdot sinc\left(\frac{a}{\pi}\omega\right)$$