EE40 Cheatsheet

Equivalent Circuits

If **no dependent** sources, turn all of the current sources into open circuits and all of the voltage sources into short circuits, and solve for the equivalent resistance ($R_{Th} = R_{eq}$).

For any circuit, deactivate independent sources, add an external source v_{ex}, and solve for i_{ex}. $R_{Th} = v_{ex}/i_{ex}$.

For any circuit, find $v_{Th} = v_{oc}$ by solving the circuit with an open circuit at the terminals. Find R_{Th} by using the fact that

$i_{sc} = \frac{V_{Th}}{R_{Th}}$

Units

$$Volt = V = \frac{kg \cdot m^2}{A \cdot s^3} = A \cdot \Omega = \frac{W}{A} = \frac{J}{C}$$
$$Ampere = A = \frac{C}{s}$$
$$Coulomb = C = F \cdot V = A \cdot s$$
$$Farad = F = \frac{A \cdot s}{V} = \frac{J}{V^2} = \frac{W \cdot s}{V^2} = \frac{C}{V}$$
$$F = \frac{C^2}{J} = \frac{C^2}{N \cdot m} = \frac{s^2 \cdot C^2}{m^2 \cdot kg} = \frac{s^4 \cdot A^2}{m^2 \cdot kg} = \frac{s}{\Omega}$$

Circuit Properties

$$M(\omega_c) = \frac{M_0}{\sqrt{2}}$$

$$B = \begin{cases} 0 \leq \omega < \omega_c & \text{lowpass} \\ \omega > \omega_c & \text{highpass} \\ \omega_{c1} < \omega < \omega_{c2} & \text{bandpass} \\ \omega < \omega_{c1} \text{ and } \omega > \omega_{c2} & \text{bandreject} \end{cases}$$

Quality Factor Q

$$Q = 2\pi \left(\frac{W_{stor}}{W_{diss}} \right) \Big|_{w=w_0}$$

Impedance Matching

Maximum Power Transfer occurs when
$$Z_L = Z_S^*$$
Minimum reflection occurs when
$$Z_L = Z_S$$

Op-Amps

- Infinite voltage gain ($G \to \infty$).
- Infinite input impedance ($R_{in} \to \infty$).
  - Implies $i_p = i_n = 0$.
- Zero output impedance ($R_{out} \to 0$).
- For negative feedback configurations only, $v_p = v_n$.
- Analyze normally, but not at output (voltage not well-defined there).

Resistors

- i, v relation $i = v/R$
- v, i relation $v = iR$
- Power $p = i^2 R$
- Stored energy $w = 0$
- Series combination $R_{eq} = R_1 + R_2$
- Parallel combination $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$
- Allows instantaneous v and i change

Capacitors

- i, v relation $i_C = C \frac{dv_C}{dt}$
- v, i relation $v(t) = v(t_0) + \frac{1}{C} \int_{t_0}^{t} idt$
- Initially acts like **short** circuit.
- $q = Cv$
- Power $p = Cv \frac{dv}{dt}$
- Stored energy $w = (1/2)Cv^2$ (should be in Joules)
- Combine **opposite** of resistors
- Series combination $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$
- Parallel combination $C_{eq} = C_1 + C_2$
- Becomes **open** if left in DC for a long time
- **Voltage** across can't change instantly ($v_0 = v_{0-}$).
- Allows instantaneous i change
- Natural response $v(t) = [v(t_0) - v(\infty)]e^{-(t-t_0)/\tau)}$
- $\tau = \frac{1}{RC}$

Inductors

- i, v relation $i_L = i(t_0) + \frac{1}{L} \int_{t_0}^{t} vdt$
- v, i relation $v_L = L \frac{di_L}{dt}$
- Initially acts like **open** circuit.
- Becomes **shorted** if left in DC for a long time
- $p = Li \frac{di}{dt}$
- Stored energy $W = \frac{1}{2} Li^2$ (should be in Joules).
- Combine like resistors
- Series combination $L_{eq} = L_1 + L_2$
- Parallel combination $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$
- **Current** across can't change instantly ($i_0 = i_{0-}$).
- Allows instantaneous v change
- Natural response $i(t) = i(\infty) + [i(t_0) - i(\infty)](e^{-(t-t_0)/\tau})$
- $\tau = \frac{1}{RL}$

RLC Circuits

Diff e.q. $x'' + ax' + bx = c$
Initially $x(0)$ and $x'(0)$
Finally $x(\infty) = \frac{c}{b}$, $\alpha = \frac{a}{2}$, $\omega_0 = \sqrt{b}$.
$\omega_c = \omega_0 = \frac{1}{\sqrt{LC}}$, $\omega_d = \sqrt{w_0^2 - \alpha^2}$

Series

$x(t) = v(t)$, $\alpha = \frac{R}{LC}$

Parallel

$x(t) = i(t)$, $\alpha = \frac{1}{2RC}$

Overdamped Response $\alpha > \omega_0$

$$x(t) = [x(\infty) + A_1 e^{s_1 t} + A_2 e^{s_2 t}]u(t)$$
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$
$$A_1 = \frac{x'(0) - s_2[x(0) - x(\infty)]}{s_1 - s_2}$$
$$A_2 = -\frac{x'(0) = s_1[x(0) - x(\infty)]}{s_1 - s_2}$$

Critically Damped $\alpha = \omega_0$

$$x(t) = [x(\infty) + (B_1 + B_2 t)e^{-\alpha t}]u(t)$$
$$B_1 = x(0) - x(\infty)$$
$$B_2 = x'(0) + \alpha[x(0) - x(\infty)]$$

Underdamped $\alpha < \omega_0$

$$x(t) = x(\infty) + [D_1 cos\omega_d t + D_2 sin\omega_d t]e^{-\alpha t}u(t)$$
$$D_1 = x(0) - x(\infty)$$
$$D_2 = \frac{x'(0) + \alpha[x(0) - x(\infty)]}{\omega_d}$$
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Bandpass

$B = \omega_{c2} - \omega_{c1} = \frac{R}{L}$ $\omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$ $Q = \frac{\omega_0}{B}$

$$\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$
$$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Series Bandpass

$B = \frac{R}{L}$ $Q = \frac{\omega_0 L}{R}$

Parallel Bandpass

$B = \frac{1}{RC}$ $Q = \frac{R}{\omega_0 L}$

Dividers

Current divider:
$$i_1 = \frac{R_2}{R_1 + R_2} i_{source}$$
Voltage divider:
$$V_{mid} = \frac{R_{ground}}{R_{ground} + R_{source}}$$

Nodal Analysis

Identify all extraordinary nodes, and choose one to be ground. Use KCL at each other node, with currents **leaving** the node. All coefficients of the voltage at the node in the resulting equation should be positive (i.e. V_1/R, not $-V_1/R$).

Supernode

If a voltage source connects two nodes, create one KCL equation for both. Also use $V_p - V_n = V_s$.

Mesh Analysis

Assign a clockwise current for each loop. Apply KVL and solve the resulting equations.

Supermesh

If a current source is between two meshes, create one KVL equation for both. Also use $i_1 - i_2 = i_{1,s}$.

$Y \to \Delta$ Transformation

R_a between $2, 3$. R_b between $1, 3$. R_c between $1, 2$.

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

$\Delta \to Y$ Transformation

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

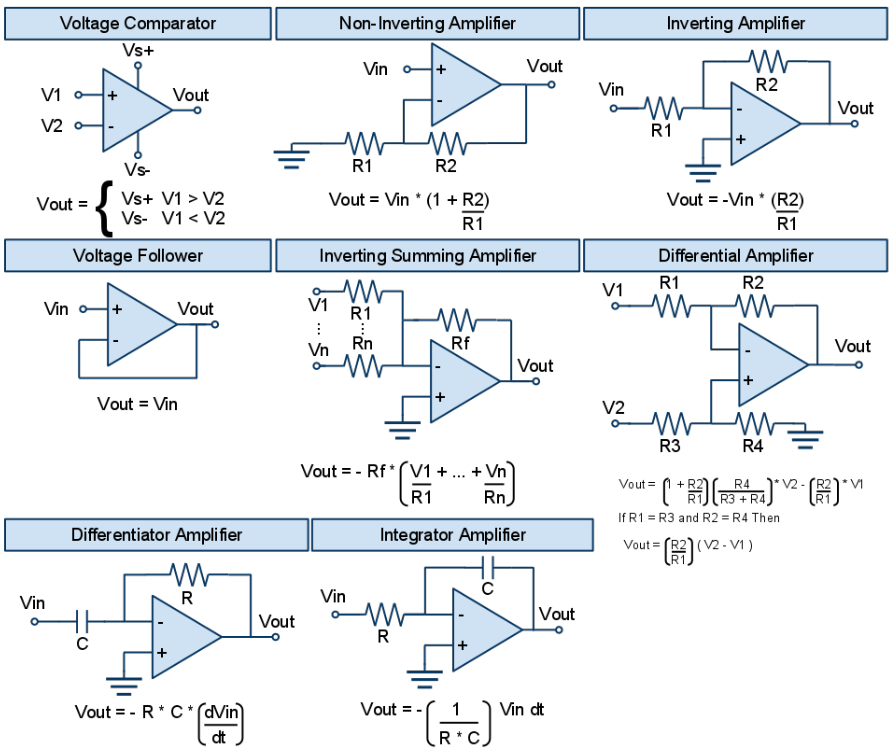
Wheatstone Bridge

$$V_{out} \approx \frac{V_0}{4} \left(\frac{\Delta R}{R} \right)$$

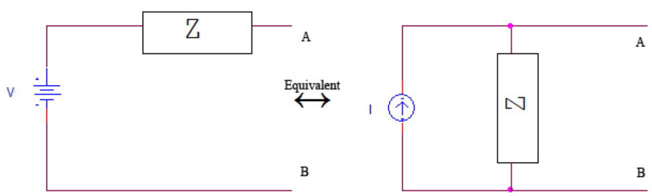
Superposition

If there are only independent sources, they can be solved in disjoint sets, with all other's disabled.

Basic Operational Amplifier Configurations

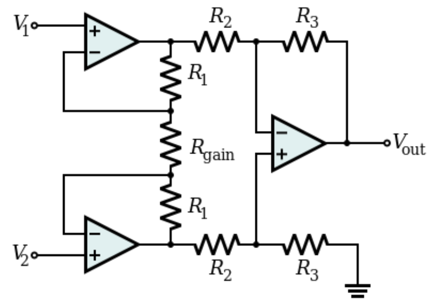


Source Transform



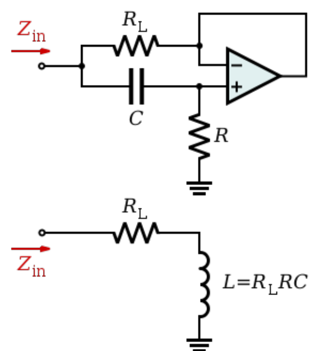
$V = IZ,\ I = \frac{V}{Z}$

Instrumentation Amplifier

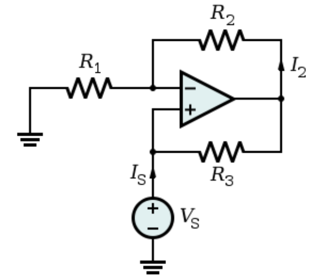


$$Gain = \frac{V_{out}}{V_2 - V_1} = \left(1 + \frac{2R_1}{R_{gain}}\right)\frac{R_3}{R_2}$$

Inductance Gyrator

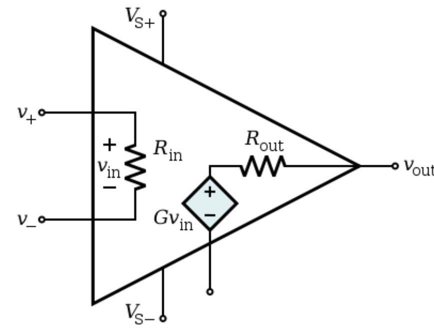


Negative Impedance Converter

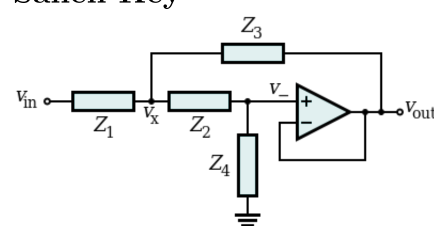


$R_{in} = -R_3\frac{R_1}{R_2}$

Non-Ideal Op-Amp



Sallen-Key



$$\frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3(Z_1 + Z_2) + Z_3 Z_4}$$

$$v_x = v_{out}\left(\frac{Z_2}{Z_4} + 1\right)$$