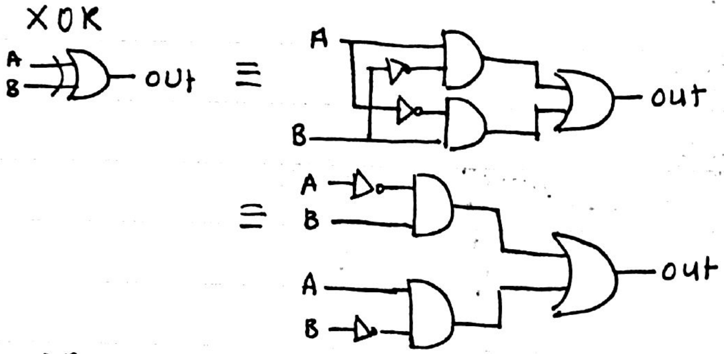
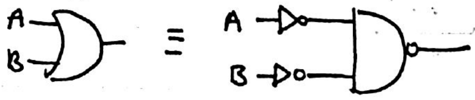
## GATES

AND OR NOT

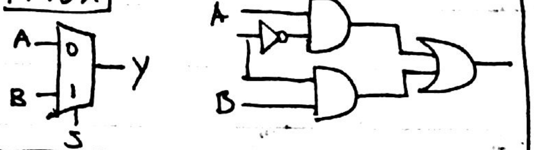

XOR



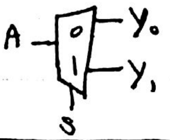

OR




## MUX




## Decorder

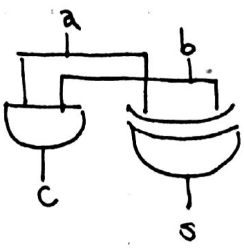



| S | $Y_0$ | $Y_1$ |
|---|---|---|
| 0 | A | 0 |
| 1 | 0 | A |

## Half adder

-adds 2-bits

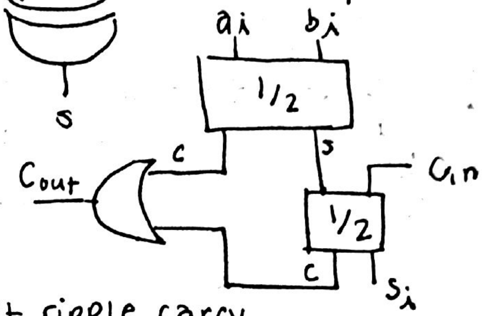
| a | b | c | s |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |




## Full adder

-adds 3bits

| a | b | c | c | s |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |




## Static power

$\sim \frac{V_{DD}^2}{R_{off}}$

Switching Power

$\tau = R_{on} \cdot C_{in}$

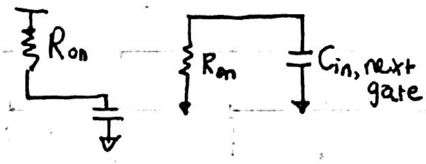
- ⇒ max clock freq

* n-bit ripple carry

worst-case delay: n * stage delay

↳ time b/w input came + output shows up

## Dynamic Power




$Q_{charge} = C_{in} \cdot V_{dd}$

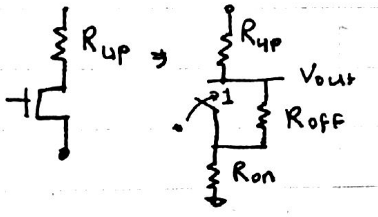
$\Delta E_{battery} = Q \cdot V_{DD} = C_{in} \cdot V_{DD}^2$

$E_{cap} = \frac{1}{2} C_{in} V_{dd}^2$

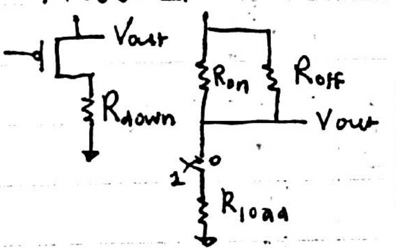
$P_{avg} = \frac{\Delta E}{T_{avg, 0\to1}} = \frac{C_{in} \cdot V_{dd}^2}{T_{avg, 0\to 1}}$

$= (f_{0\to1})(C_{in})(V_{dd}^2)$

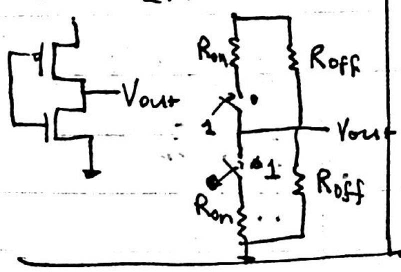
## NMOS invertor (INVN)



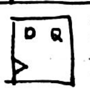

## PMOS: INVP




## CMOS: INVC

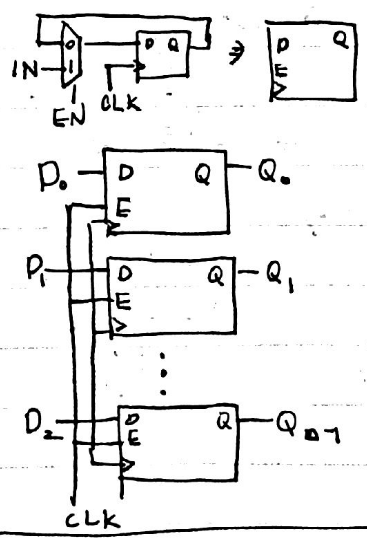



## Edge-triggered D-ff




## MxN register file

D-ff w/ enable



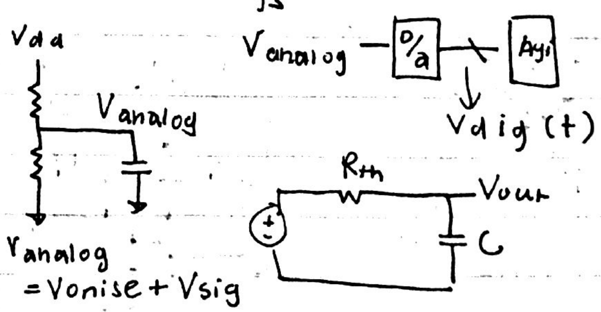

## sampling

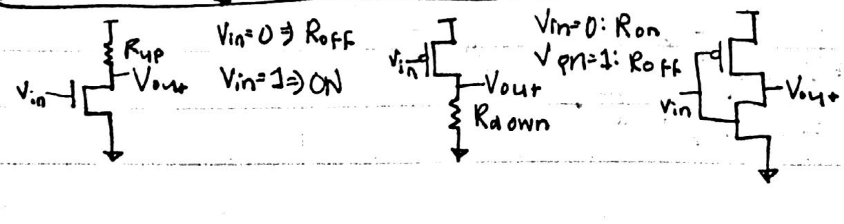
$f_{min} = 2 \cdot$ freq of signal $= f_{nyquist}$

$f_{nyquist} = \frac{f_{sample}}{2}$

$W_p = 2\pi \cdot f_{nyquist} = \frac{1}{R_{th} \cdot C}$

$t_s = n \cdot T_s = n \cdot \frac{1}{f_s}$




$V_{in} = 0 \Rightarrow R_{off}$

$V_{in} = 1 \Rightarrow$ ON

$V_{in} = 0$: $R_{on}$

$V_{in} = 1$: $R_{off}$

$V_{in} = 0$: $R_{on}$ ON TOP, $R_{off}$ ON Bottom

$V_{in} = 1$: $R_{on}$ ON Bottom, $R_{off}$ ON TOP

## HW 8

### Representing Bits

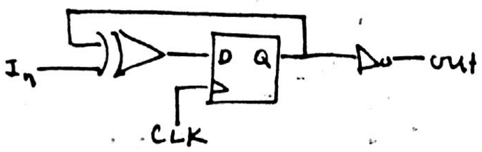
$2^{10} \approx (10^3) \rightarrow 10 \text{ bits}$

$10^6 \rightarrow (10^3)^2 \rightarrow (2^{10})^2 \rightarrow 20 \text{ bits}$

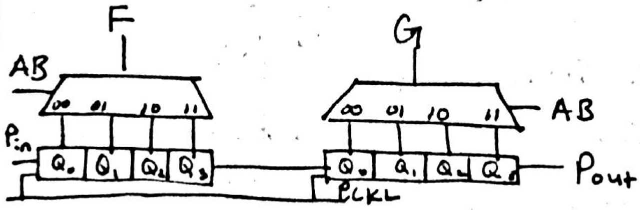
$10^9 \rightarrow (10^3)^3 \rightarrow (2^{10})^3 \rightarrow 30 \text{ bits}$

$3600 \rightarrow 2^2 \cdot 10^{10} \rightarrow 12 \text{ bits}$

### Finite State Machine

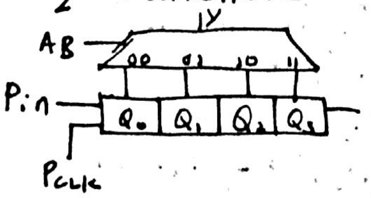



### Artificial GATE

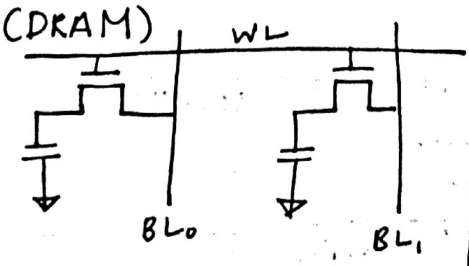



### FPGA: Reconfigurable GATE

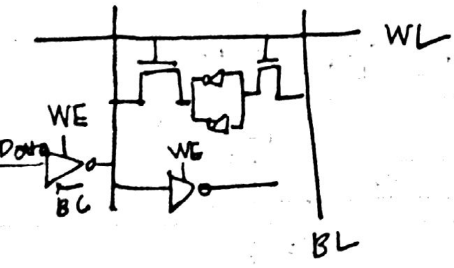
$2^{2^n}$ functions



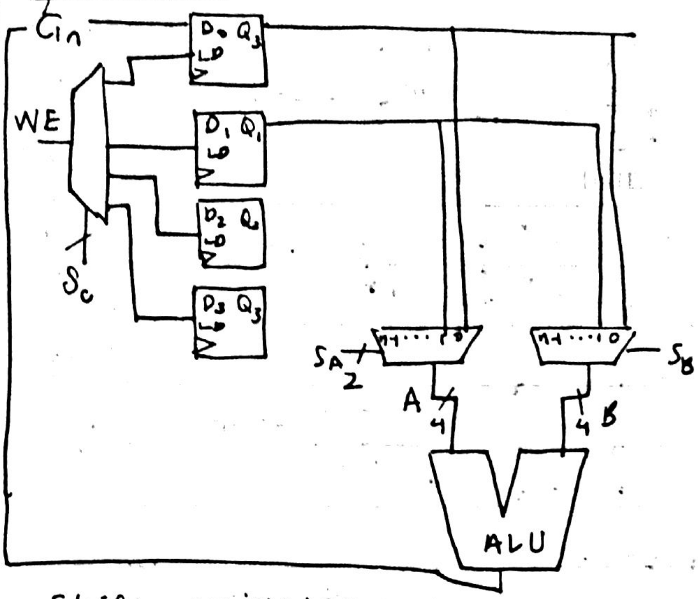

### Dynamic Random Access Mem (DRAM)



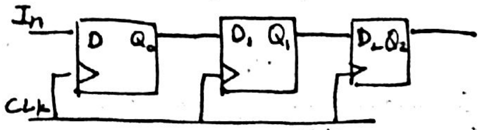

### Static RAM



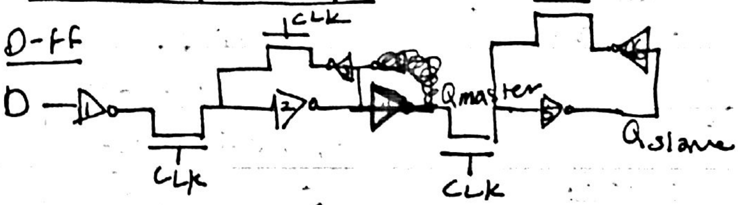

### Register file

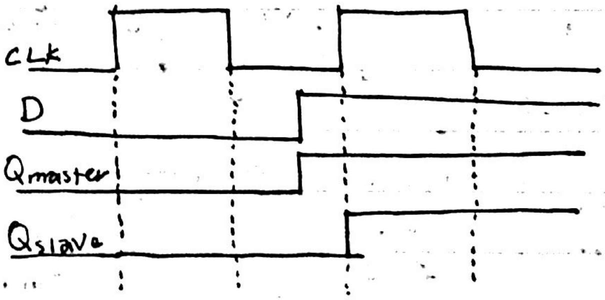



### Shift resigster

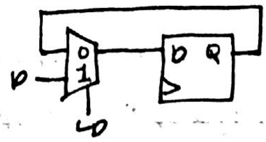



### D-FF

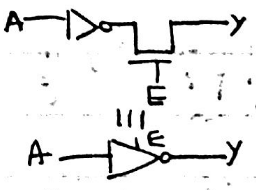


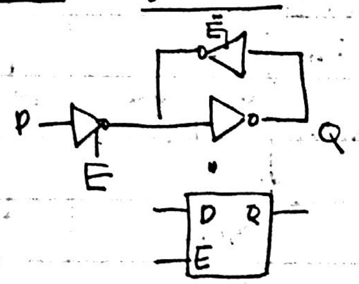

### D-ff w/Enable



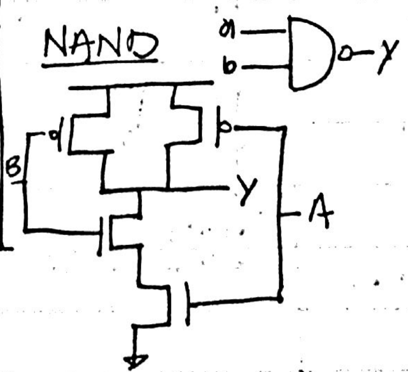

### Enabled Inverters



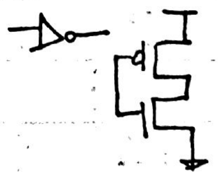

### D-latch



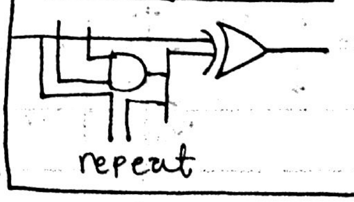

### NAND



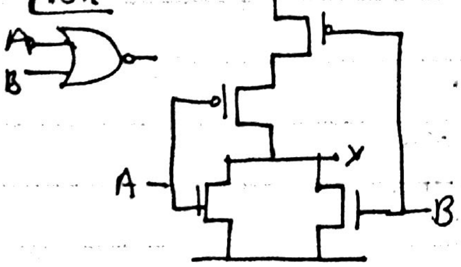

### Invertor



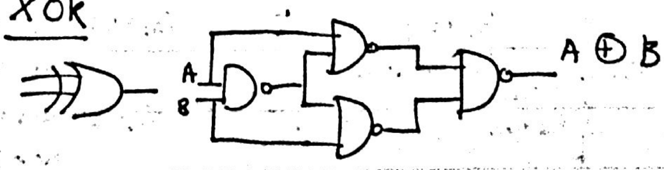
### +1 block (HW 11)




### NOR




### XOR



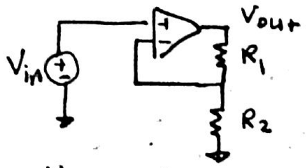
## Op-Amps

Ideal: $i_n = i_p = 0$

$R_i = \infty$, $A = \infty$, $R_o = 0$

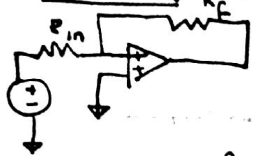
$V_o = A(V_+ - V_-) = 10^6$

### Non-inverting



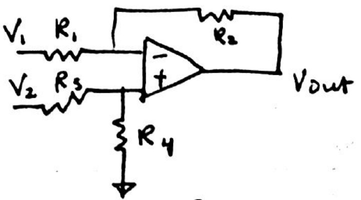

$$\frac{V_{out}}{V_{in}} = \frac{R_1+R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

### Inverting



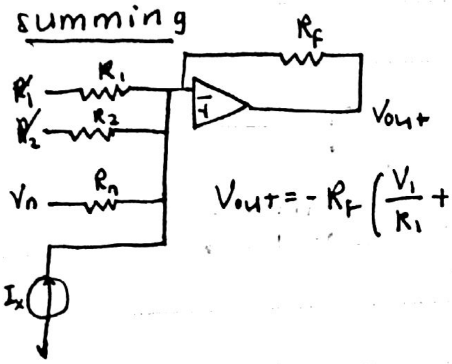

$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

### Subtractor




$$V_{out} = -V_1 \frac{R_2}{R_1} + \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)V_2$$

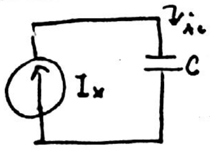
### Summing




$$V_{out} = -R_f\left(\frac{V_1}{R_1} + \dots + \frac{V_n}{R_n}\right) + I_x$$

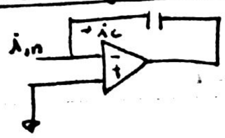
### Integrator

$i_o = C\frac{dV_c}{dt} = i_x$




$\frac{dV_c}{dt} = \frac{1}{C} i_x$

### Inverting w/ capacitor

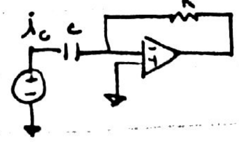



$i_c = i_x$

$V_c(t) = \frac{1}{C}\int_0^t i_x dt$

$V_o = -V_c = -\frac{1}{C}\int_0^t i_x dt$

### Differentiator




KVL: $V_c = V_i$

$i_c = \frac{0 - V_o}{R}$

## Approximations

$I_s = 10^{-15}A = 1fA$

forward biase b/c $V_D > 0$

$I_d(0.6V) = (10^{-15}A)(10^{0.6/60mV}) = 10^{-5}A$

$I_d(0.66V) = 10^{-4}A = 0.1mA$

$I_d(0.72V) = 1mV$

$I_d(0.626) = 27\mu A$

$e^x = 10^{x/\ln 10}$

→ current iff $V_D > \sim 0.7V$

if diode on: —•—•—

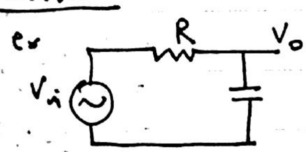
if diode off: —o o—

$I_D(V_D) = I_s(e^{V_D/V_{th}} - 1)$

$$\approx \begin{cases} -I_s & V_D < 100mV \\ 0 & V_D = 0 \\ I_s e^{V_D/V_{th}} & V_D > 100mV \end{cases}$$

$\Rightarrow I_D = I_s 10^{V_D/60mV}$ at R.T

### Phasor




$V_o = V_i - C\frac{dV_o}{dt}R$

### Magnitude

$$\left|\frac{a+bj}{c+dj}\right| = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$$

$$\angle\frac{a+bj}{c+dj} = \angle a+bj - \angle c+dj$$

### Complex #s

Rectangular form:

$n = a + jb$

$c = \sqrt{a^2+b^2}$

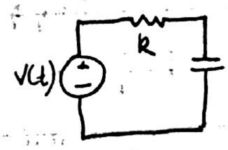
$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

polar form:

$n = c e^{j\theta} = c\angle\theta$

$c$ ↑ mag, $\theta$ phase

### Steady State




$V(t) = d\cos(\omega t + \phi)$

$\cos(\omega t + \phi) = -\sin\phi\sin(\omega t) + \cos\phi\cdot\cos(\omega t)$

$V_{in}(t) = d(\cos\phi\cdot\cos(\omega t) - \sin\phi\sin(\omega t))$

$= X\cdot\cos\omega t - Y\sin(\omega t)$

$V_{out}(t) = \beta\cos(\omega t + \phi_{out})$

$= U\cos(\omega t) - Z\sin(\omega t)$

## Zseries

① $Z_{series} = R + \frac{1}{j\omega C} = \frac{1 + j\omega RC}{j\omega C}$

$= \frac{1 + j\omega/\omega_p}{j\omega C}$ ← zero, ← pole

$$= \begin{cases} \frac{1}{j\omega C} & \omega \ll \omega_p \\ \frac{1+j}{j\omega_p C} & \omega = \omega_p \\ R & \omega \gg \omega_p \end{cases}$$

② $|Z_{series}| = \begin{cases} \frac{1}{\omega C} & \omega \ll \omega_p \\ \sqrt{2}/\omega_p C & \omega = \omega_p \\ R & \omega \gg \omega_p \end{cases}$

③ $\angle Z_{series} = \begin{cases} -90 & \omega \ll \omega_p \\ -45 & \omega = \omega_p \\ 0 & \omega \gg \omega_p \end{cases}$ $\leftarrow \frac{1+j}{j}$

### Z parallel

$Z_{parallel} = \frac{Z_R Z_C}{Z_R + Z_C} = \frac{R}{j\omega C}\frac{1}{R + \frac{1}{j\omega C}} = \frac{R}{j\omega RC + 1}$

$= \frac{R}{1 + j\omega/\omega_p}$

$$= \begin{cases} R & \omega \ll \omega_p \\ \frac{R}{1+j} & \omega = \omega_p \\ \frac{1}{j\omega C} & \omega \gg \omega_p \end{cases}$$

① $|Z_{parallel}| = \begin{cases} R & \omega \ll \omega_p \\ R/\sqrt{2} & \omega = \omega_p \\ 1/\omega C & \omega \gg \omega_p \end{cases}$

② $\angle Z_p = \begin{cases} 0 & \omega \ll \omega_p \\ -45 & \omega = \omega_p \\ -90 & \omega \gg \omega_p \end{cases}$

$$\boxed{|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_p)^2}} \qquad \angle H(j\omega) = -\tan^{-1}(\omega/\omega_p)}$$

## Root Mean Square

$$\begin{cases} V_{RMS} = \frac{V_{op}}{\sqrt{2}} \\ V_{pp} = 2V_{op} \\ V_{op} = \sqrt{2}\cdot V_{RMS} \end{cases}$$

$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt = \frac{V_{op}^2}{2R}$

$V_{avg} = \frac{1}{T}\int_0^T \sin(\omega t)\,dt = 0$

### tan approx

$\tan^{-1}(\pm 1) = \pm 45°$, $\tan^{-1}(0) = 0$

$\tan^{-1}(x) = x$ $x \ll 1$, $\tan^{-1}(x) = \frac{\pi}{2}$ if $x \gg 10$

| $\frac{V_o}{V_i}$ | $\frac{P_o}{P_{in}}$ | dB |
|---|---|---|
| 1 | 1 | 0 |
| 10 | $10^2$ | 20 |
| 1/10 | $10^{-2}$ | -20 |
| 2 | 4 | 6 |
| $\sqrt{2}$ | 2 | 3 |
| $1/\sqrt{2}$ | 1/2 | -3 |

$$\frac{P_o}{P_i} = 10\cdot\log_{10}\left(\frac{V_o}{V_i}\right)^2 = 20\log_{10}\left(\frac{V_o}{V_i}\right)$$

$$r[dB] = 10\,r[B] = 10\log_{10}\frac{P}{P_{ref}}$$

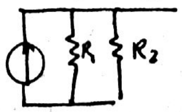
① $1dB = 10\log_{10}\left(\frac{P}{P_{ref}}\right)$

② $10\log(2) = 3dB \rightarrow 10^{\frac{3}{10}} = 2$

③ $\sqrt{10} \approx 3.15 \rightarrow 10^{1/2} = 3.5 \rightarrow 2dB$

④ $2dB = 10\log(1.6)$

## current divider




$$I_2 = I\left(\frac{R_1}{R_1+R_2}\right)$$

$$I_1 = I\left(\frac{R_2}{R_1+R_2}\right)$$

## Inductor

$$V_L = L\frac{di}{dt}$$

$$W = \frac{1}{2}Li^2$$

$$P = IV = L\frac{di}{dt}\,i$$

## capacitors

$$V = E.l$$

$$Q = C_{eq}V_{total}$$

$$I_c = C\cdot\frac{dV_c}{dt}$$

$$V_c(t) = V[t_0] + \frac{1}{C}\int_{t_0}^{t} I_c\,dt$$

$$W = \frac{1}{2}CV^2$$

$$P = IV = C\cdot\frac{dV}{dt}\cdot V$$

$$I(t) - I_0 = m(t - t_0)$$

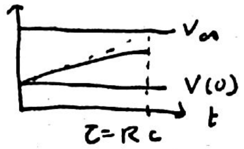
$$V_{in}(t) = |V_i|\cos(wt + \varnothing)$$

$$V_{out}(t) = |H(jw)|\,|V_{in}|\cos(wt + \varnothing_i + \angle H(jw))$$

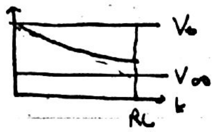
## transients

① $V(0) < V_\infty$

$$V_{out} = V_\infty + \underbrace{[V(0) - V_\infty]e^{-t/RC}}_{\doteq\, e^{-1} = 0.37}$$




② $V(0) > V_\infty$




$$V_{out} = V_\infty + [V(0) - V_\infty]e^{-t/RC}$$

$$= V_\infty + [V(0) - V_\infty](1 - t/RC)$$

$$= [V(0) - V_\infty](-t/RC) + V(0)$$

$$\text{slope} = -\frac{[V(0) - V_\infty]}{RC}$$

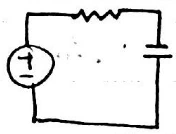
## Quality factor

$$Q = \frac{\text{Energy stored}}{\text{Energy dissipated/cycle}}$$

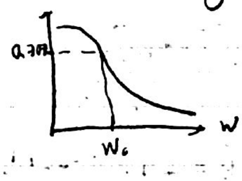
$$= \frac{\text{Energy stored}}{\frac{\text{Energy dissipated}}{\text{sec}}\cdot\frac{1}{W_0}\left[\frac{\text{cycle}}{\text{sec}}\right]}$$

$$= W.\frac{L}{R}$$

## lowpass filter



$$V_c = \frac{V_s + Z_c}{R + Z_c},\ \frac{V_s/jwc}{R + \frac{1}{jwc}}$$




| $\frac{V}{V_0}$ | $\frac{P}{P_0}$ | dB | |
|---|---|---|---|
| 1 | 1 | 0 | |
| 10 | $10^2$ | 20 | 20 dB / decade |
| $10^2$ | $10^4$ | 40 | |
| $10^{-1}$ | $10^{-2}$ | -20 | -20 dB / decade |
| $10^{-2}$ | $10^{-4}$ | -40 | |
| 2 | 4 | 6 | 6 dB / octave |
| 4 | 16 | 12 | |
| 8 | 64 | 18 | |

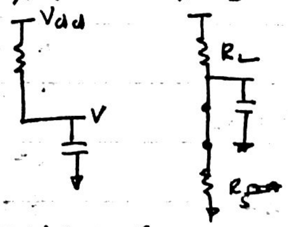
## Digital circuit

$$\text{Freq} = f = \frac{1}{T}$$


$V(nT) = 0 \Leftrightarrow X_n = 0$

$V(nT) = V_{dd} \Leftrightarrow X_n = 1$

within 20% accuracy




## LC TANK in parallel & series

① $Z_p = \frac{Z_L Z_C}{Z_L + Z_C} = \frac{jwL}{jwC}\cdot\frac{1}{\left(\frac{1}{jwC} + jwL\right)} = \frac{jwL}{1 - w^2LC}$

② $Z_0 = R + jwL + \frac{1}{jwC} = R + \frac{1 - w^2LC}{jwC}$

③ $Q = \frac{W_0 L}{R} = (W_p C R)^{-1} = \frac{1}{W_p C R}$

$$W_p = \frac{1}{\sqrt{LC}}$$

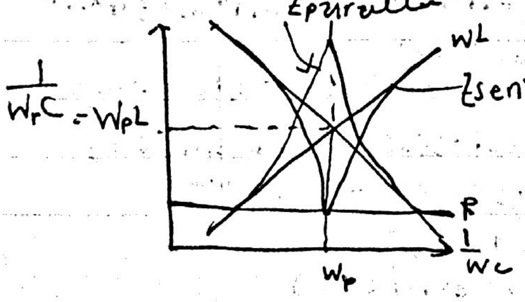
④ $|Z_{tank}(jW_p)| = Q^2R = Q(W_pL) = Q\left(\frac{1}{W_pC}\right)$

① n bits ⇒ $2^n$ states $(y_0, y_1)$

② n states ⇒ $\log_2 n$ bits

① $W_p = 2\pi f = \frac{1}{RC}$

$f_x < \frac{f_s}{2}$, $f_x \approx \frac{f_s}{2}$, $T_x = 2T_s$, $T_x = 2T_s$




* conductivity, $\sigma = \frac{1}{\rho}$

$$\rho\frac{L}{A} = R$$

$f_x < \frac{f_s}{2}$

$T_x = 2T_s$

* $\forall_n$ sampling

$$V(t) = \cos(2\pi(f_s + \delta f)nT_s)$$

$$= \sin\left[2\pi n + 2\pi\frac{\delta f}{f_s}n\right]$$