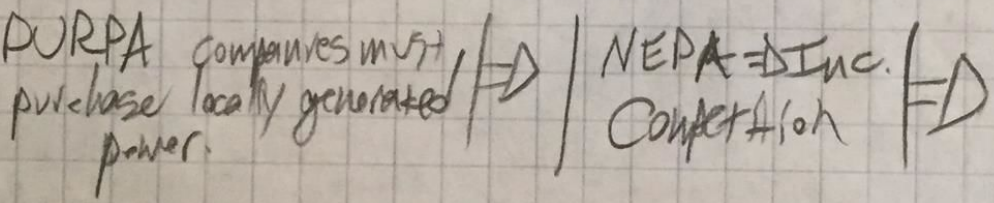
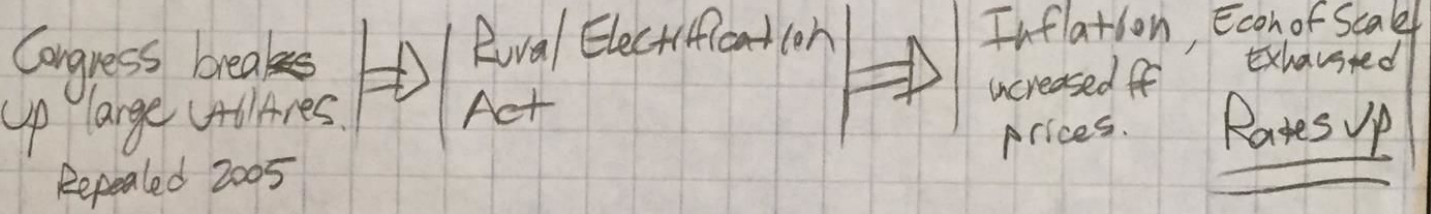
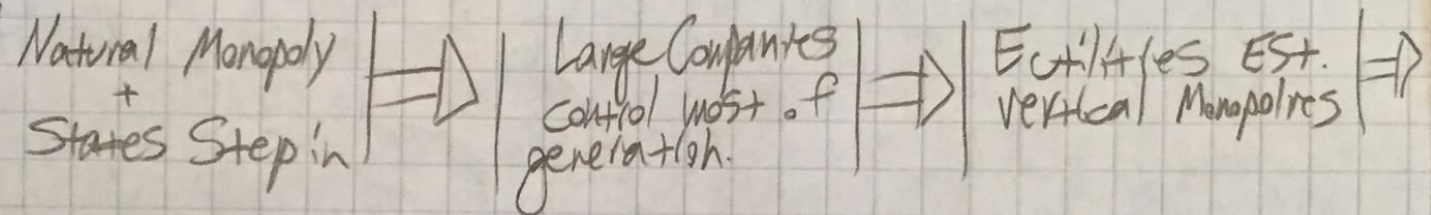
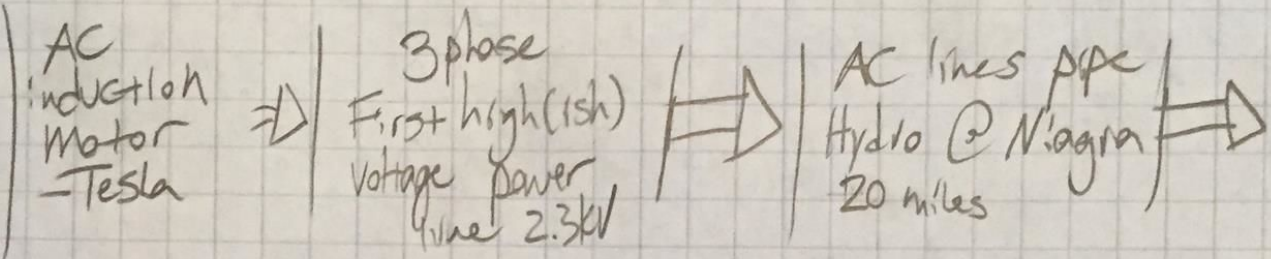
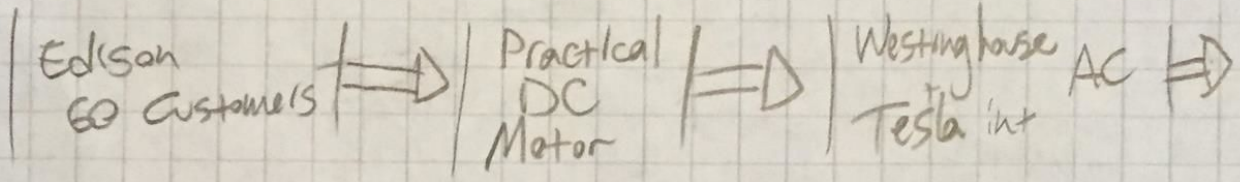
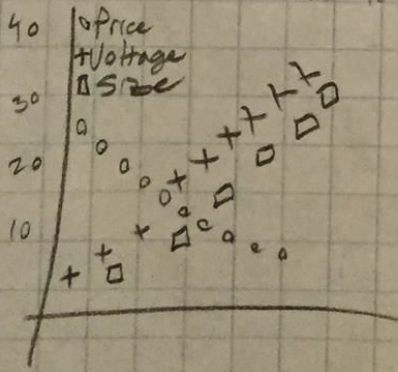


# EE 137A MTI Review



## Natural Monopoly

- Stems from
- High Cap. costs
  - Econ of Scale
  - Barriers to M. Entry
- } more efficient for 1 large company.



## Vertical Monopoly

- Transmission
- Generation
- Distribution
- Customer Service.

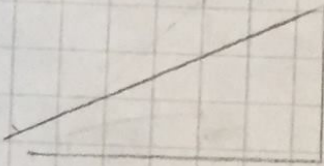
## Legacy Grid

- One-way, Radial Power flow
- Redundancy
- Stability through Inertia

## Smart Grid

Higher Spatiotemporal Resolution

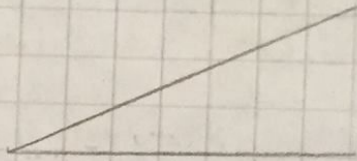
# Complex Power



$$Z = R + jX$$

$$R = Z \cos \theta$$

$$\cos \theta = \text{p.f.}$$



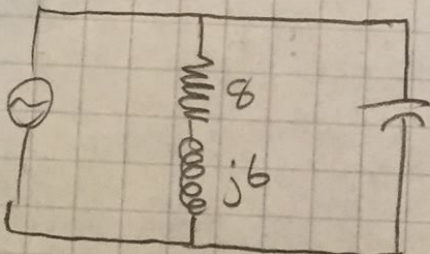
$$S = P + jQ$$

$$P = S \cos \theta$$

$$V = |V| e^{j\theta}$$

$$e^{j\theta} = \cos \theta + j \sin \theta$$

Parallel RC circuit



Power into Each Branch?

$$S_1 = 450W + 337.5VAR$$

$$S_2 = 0W - 937.5VAR$$

$$S = S_1 + S_2 = 450W - 600VAR$$

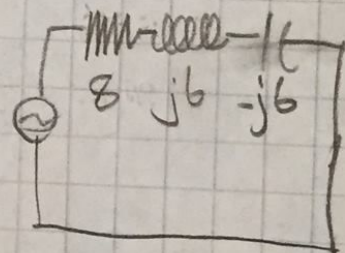
$$\text{P.f.}_1 = .8 \text{ lagging}$$

$$\text{P.f.}_2 = 0$$

$$\text{P.f.}_{\text{tot}} = .6 \text{ leading}$$

$$V = 75 \angle 0^\circ$$

$$V(t) = 75\sqrt{2} \cos \omega t$$



## Why 50-60 Hz?

Too Low: Noticeable flicker

Too High: Reactive losses

## Why 3 phases

• Steady Torque

• Wire can be neglected or minimized.  
Current sums to zero.

## Load Types

Resistive: Lamps  
Heaters

Inductive: Motors  
Power tools  
Appliances

Capacitive: Arc welder

## Inductor

E in Field (B)

Likes low Hz (DC)

Resists change in I

Current lags Voltage

Inductance L

$$X_L = \omega L$$

$$E_L = \frac{1}{2} LI^2$$

## Capacitor

• Stores E charge

• Likes high Hz also (AC)

• Resists changes in V

• AC lags voltage

Farads, C

$$X_C = \frac{1}{\omega C}$$

$$E_C = \frac{1}{2} CV^2$$

Inductance  
Usually dominates  
So power factor is  
usually lagging.

Real Power:

$$P_{ave} = I_{RMS} V_{RMS} \cos \theta$$

Reactive

$$S_{ave} = I_{RMS} V_{RMS} \sin \theta$$

Complex:

$$\underline{S} = \underline{I}^* \underline{V}$$

$$Z = R + jX$$

Impedance = Resistance  
+ Reactance

Admittance = Conductance + Susceptance

$$Y = G + jB$$

$$Y = \frac{1}{Z}$$

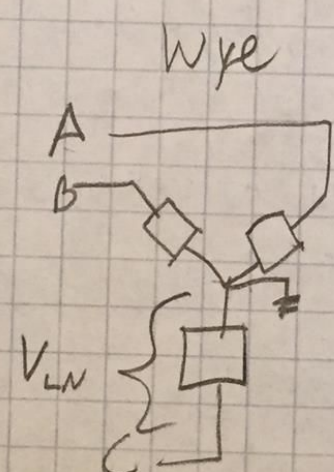
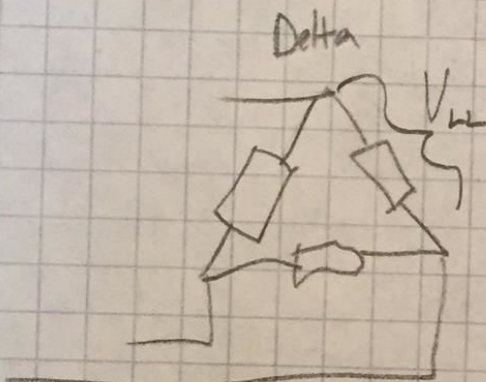
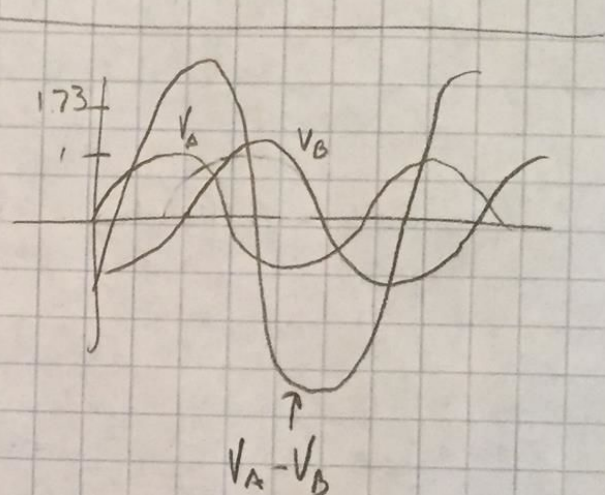
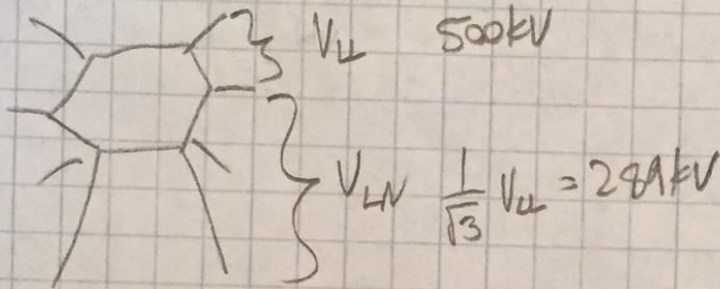
### L3] Why is Reactive Power A problem?

Line losses associated with circulating current.

Capacity limitations dictated total current.

### Why Shunt and not series capacitors

- fault current
- bypass switch/protection
- Vulnerability to Resonance



Also phase Angle is Shifted  $30^\circ$

$$V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_{an} \angle 30^\circ$$

$$\underline{V_{LL} = \sqrt{3} V_{LN} \text{ (magnitude)}}$$

$$S_{3\phi} = |S_{3\phi}| = 3V_{LN} I_L = \sqrt{3} V_{LL} I_L \text{ VA}$$

$V_{LL}$  higher,  
 $\sqrt{3} V_{LN}$

Legacy Grid

- one-way, radial power flow
- Redundancy
- Stability through Inertia



Smart Grid

- Increased sensors allow for higher spatio-temporal resolution.

Natural Monopoly

- Stems from
- High Cap. costs
  - Econ. of Scale
  - Barriers to market entry
- more efficient for large company

Vertical Monopoly

- Transmission • Distribution
- Generation • Customer Service

Why 50-60Hz

- Too Low: Noticeable Flicker
- Too High: Reactive Losses

Why 3 phases

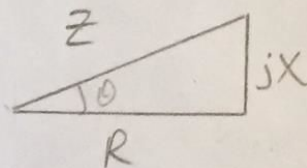
- Steady Torque
- Sum of 3 phase current is zero, so neutral return can be neglected.

Load Types

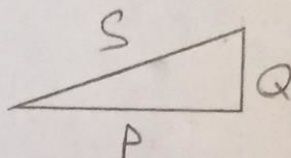
- Resistive: Lamps, Heaters
- Inductive: Motors, power tools.
- Capacitive: Arc Welders

$Z = R + jX$	$Y = G + jB$
Impedance	Admittance
Resistance	Conductance
Reactance	Susceptance
	$Y = \frac{1}{Z}$

Complex Power



$R = Z \cos \theta$     $\cos \theta = \text{p.f.}$

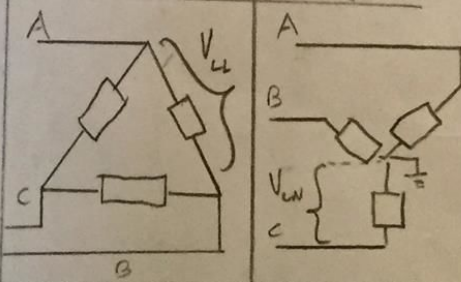
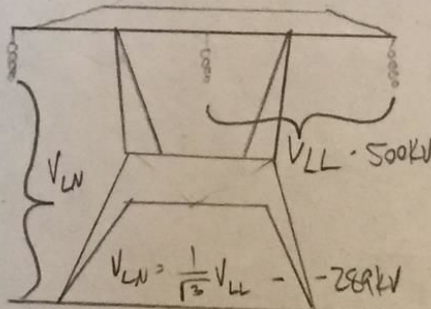


$P = S \cos \theta$

$V = |V| e^{j\theta}$   
 $e^{j\theta} = \cos \theta + j \sin \theta$

Reactive power is bad, why?

Line losses associated w/ circulating current  
 Capacity limitations dictated by total current.



$V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_{an} \angle 30^\circ$   
 phase angle shifted 30°  
 $|S_{3\phi}| = 3 V_{LN} I_L = \sqrt{3} V_{LL} I_L \text{ VA}$

Symmetrical Components

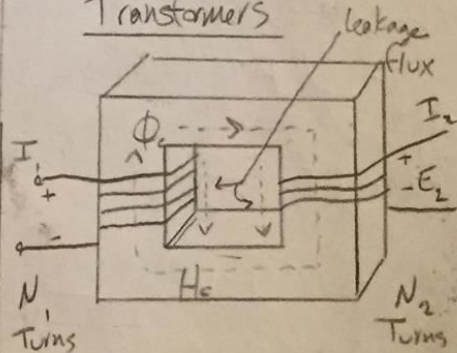
Max Morrison

$V_{abc} = \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$

Nonlinear loads cause current harmonics  
 Cause voltage harmonics  
 Total P.f. = Displacement p.f. + Distortion p.f.

Transformers



In core:  $B_c = \mu_c H_c$   
 Magnetic flux  $\phi_c = B_c A_c$   
 Reluctance  $R_c = \frac{\text{path length}}{\mu_c A_c}$   
 [Kinda Resistance]  
 $N_1 I_1 - N_2 I_2 = R_c \phi_c$

Losses

- Copper - coil;  $I^2 R$
- Iron - Core
- Hysteresis
- Eddy Currents
- Stray Losses
- Vibrations

# Transmission 2

- Resistance depends on
- Specific conductivity
  - cross section
  - Temperature ~ 10%
  - spiral ~ 1-2%
  - Fix 2%

$$R = \rho \frac{l}{A}, \text{ Area: } 1 \text{ cmil} = 5.067 \times 10^{-4} \text{ mm}^2$$

Why bundled conductors?

More surface area  $\rightarrow$  cooling  
 $\rightarrow$  less corona loss

Greater GMR  $\rightarrow$  less inductance

Line Inductance  $L_x = \frac{\mu_x}{I} = 2 \times 10^{-7} \ln \left( \frac{D_c}{r'} \right) \left[ \frac{H}{m} \right]$

Equivalent Radius  $r' = e^{-1} r = .7788r$

small  $r'$  = more inductance b/c flux density  
 big  $D$  = more inductance b/c less cancellation

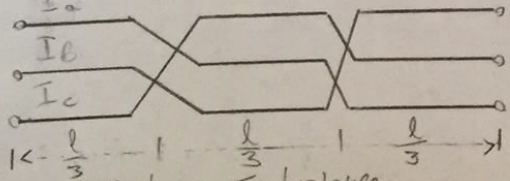
Deq =  $\sqrt[3]{D_{12} D_{23} D_{31}}$  = GMD for 3 conductors

$r'$  can also be GMR for conductor bundle

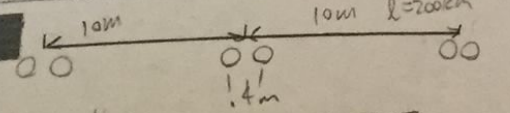
$D_{1-2} = \sqrt{(D_s \times d)^2 + d^2}$   $D_{1-3} = \sqrt{(D_s \times d)^2 + d^2}$

$D_{4\text{ cond}} = \sqrt[4]{(D_s \times d \times d \times 12d)}$

## Transposed Conductors



Balances Inductance



From table:  $GMR = .9114 \rightarrow D_{eq} = \sqrt{(0.114)(1.4)} = .0676$

Deq =  $\sqrt[3]{(10 \times 10 \times 20)}$

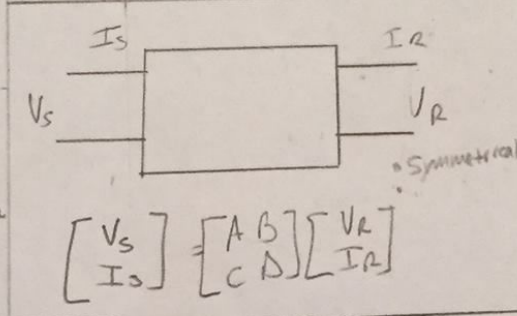
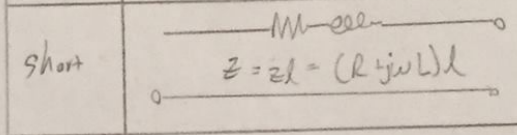
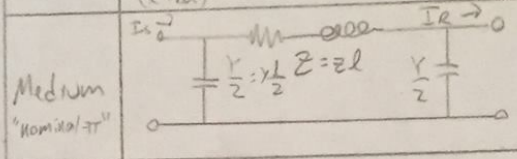
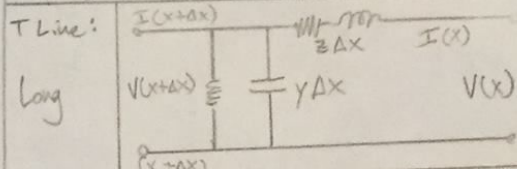
$L_a = 2 \times 10^{-7} \ln \left( \frac{12.6}{.0676} \right) (200,000) =$

$.209 \text{ H } X_a = 2\pi f L_a = 78.8 \Omega$

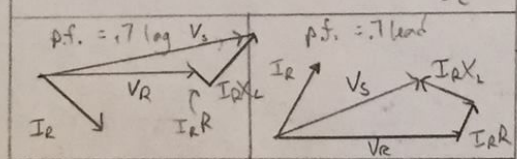
## Capacitance

$C_{xy} = \frac{\pi \epsilon}{\ln \left( \frac{D}{r} \right)}$

# EE 137 A Final Cheat Sheet



P	A = D	B	C
UNAS	P.V.	Ω	S
Short	1	Z	0
Med	$1 + \frac{YZ}{2}$	Z	$Y(1 + \frac{YZ}{4})$
Long	$\cosh(\gamma L) = 1 + \frac{\gamma^2 Z^2}{2}$	$Z_c \sinh(\gamma L) = Z$	$\frac{1}{Z_c} \sinh(\gamma L) = Y(1 + \frac{\gamma^2 Z^2}{4})$
Lossless	$\cos(\beta L)$	$j Z_c \sin(\beta L)$	$j \frac{\sin(\beta L)}{Z_c}$



Long Line Voltage Eqn

$$V(x) = k_1 e^{\gamma x} + k_2 e^{-\gamma x}$$

$$V_x = (k_1 + k_2) \frac{(e^{\gamma x} + e^{-\gamma x})}{2} + (k_1 - k_2) \frac{(e^{\gamma x} - e^{-\gamma x})}{2}$$

$$V_x = k_1 (\cosh(\gamma x)) + k_2 \sinh(\gamma x)$$

Gamma  $\gamma = \sqrt{ZY} = \alpha + j\beta$  Attenuation

Power Transfer

$$P_R = \frac{|V_s||V_r|}{|B|} \cos(\beta - \delta) = \frac{|A||V_r|^2}{|B|} \cos(\beta - \delta)$$

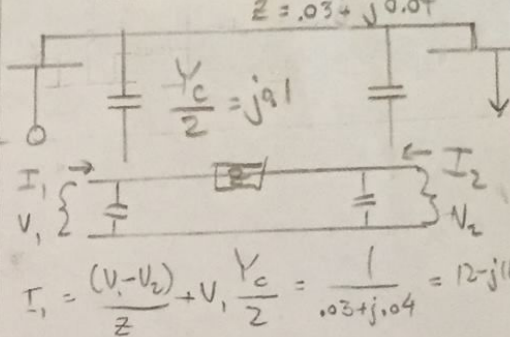
$$Q_R = \frac{|V_s||V_r|}{|B|} \sin(\alpha - \delta) = \frac{|A||V_r|^2}{|B|} \sin(\alpha - \delta)$$

T-Line Loading Limits: (short) (long lines)  
 Thermal - Expansion  $\rightarrow$  sag  
 Angle - max P @  $\delta_1 - \delta_2 = 90^\circ$ , practical @  $85^\circ$   
 Voltage - can be addressed w/ reactive comp.  $\left( \frac{\Delta V}{V} \right)$

## Power Flow

- P, Q given. Find |V|, θ
- Steady state tool, nonlinear
- Admittance matrix, (ie not Resistance)
- off diag. branches X(-1)

### 2 bus Example



$$I_1 = \frac{(V_1 - V_2)}{Z} + V_1 \frac{Y_c}{2} = \frac{1}{.03 + j0.04} = 12 - j16$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Power:  
 $S_1 = V_1 I_1^*$   
 $S_2 = V_2 I_2^*$

Usual knowns: Load Bus: P, Q  
 Gen Bus: P, V

Slack Bus; Unknown: P, Q

\* Power Balance Eqn \*

$$S_i = V_i \sum_{k=1}^n Y_{ik}^* V_k^* \left( e^{j\theta_i} = \cos\theta + j\sin\theta \right)$$

$$S_i = \sum_{k=1}^n |V_i||V_k| e^{j\theta_i - \theta_k} (G_{ik} - jB_{ik})$$

$$P_i = \sum_{k=1}^n |V_i||V_k| (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik})$$

$$Q_i = \sum_{k=1}^n |V_i||V_k| (G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik})$$

P depends on Voltage Angle b/c  $\sin(0^\circ) = 0$  and  $G$ 's small ( $R$ 's large)

Q depends on Voltage Magnitude b/c  $\cos(90^\circ) \approx 1$

Decoupled Power Flow.

$$J = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix}$$

Dishonest N-R - don't update J  
 Fast Decoupled - Simple J, no updates  
 "DC" - no reactive power,  $V = p.u.$   
 Gauss Seidel - Gauss Elimination. Small numerical iterations.

Generators

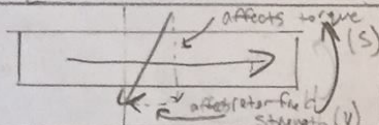
60 sec 2 poles  
min lag.

$n = \frac{1}{2\pi} f \leftarrow \text{elec}$   
mech  $P$

Increase  $P_{gen}$ : More fuel  $\rightarrow$  angle pulls ahead until  $T_{mech} = T_{prime} \rightarrow$  more power  
How to?  $\rightarrow$  more power

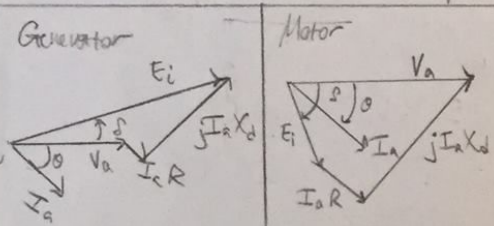
Rechanges  $\rightarrow$  Oustator changes w/it. Oustator. Field changes str. Voltage changes

When serving } inductive load  $\rightarrow$  increase excitation  
} capacitive load  $\rightarrow$  decrease



Induction Machines

- Slip causes torque through induced AC currents in rotor.
- Can't start w/o External AC power.
- Rotor lags stator, consumes reactive power



$E_i = V_a + I_a R + j I_a X_d$   $V_a = E_i - I_a R + j I_a X_d$   
overexcited - supplying Q  
under - absorbing Q

Power to Phase A  
 $S = V_e I_a^*$   
 $I_a = \frac{|E_i| \angle \delta - |V_e|}{j X_d}$   $I_a^* = \frac{|E_i| \angle -\delta - |V_e|}{-j X_d}$   
 $S = P + jQ = V_e I_a^* = \frac{|V_e| |E_i| \angle (\delta - \theta) - |V_e|^2}{-j X_d}$   
 $= \frac{|V_e| |E_i| (\cos \delta - j \sin \delta) - |V_e|^2}{-j X_d}$

$P = \frac{|V_e| |E_i| \sin \delta}{X_d}$   $Q = \frac{|V_e|}{X_d} (E_i \cos \delta - |V_e|)$

Stability

Steady State

- $\delta_{12}$  w/in limits?
- Voltages close to nominal?
- What happens w/ displacement

Transient

- Return to Eq. after contingency.

Dynamic

- Response to continuous Disturb.

Gen. Mech. Model  $\left( \frac{d(\text{in rad})}{dt} \right)$

$P_m - P_e(\delta) = J X_m \omega_s^2 = J \omega_s^2 \ddot{\delta}$   
 $\frac{P_m - P_e(\delta)}{S_B} = \frac{J \omega_s^2}{2 S_B} \frac{1}{\pi f_s} \ddot{\delta}$

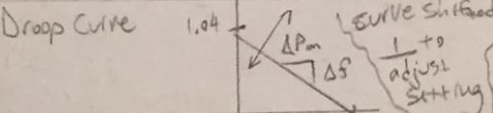
Define  $H = \frac{J \omega_s^2}{2 S_B}$  (sec or  $\frac{MJ}{MVA}$ )

$P_m - P_e(\delta) = \frac{H}{\pi f_s} \ddot{\delta}$ ,  $M = \frac{H}{\pi f_s}$

$P_m - P_e(\delta) = M \ddot{\delta}$  Damping.

Swing Eqn.

$P_m - P_e(\delta) = M \ddot{\delta} + D \dot{\delta}$



Slope =  $-R = \frac{\Delta f}{\Delta P_m} = -0.04 \text{ p.u.}$   
Droop Setting  $\rightarrow$

Area freq. Response characteristic  $\beta = \frac{1}{R} + \frac{1}{R_n}$

Droop curves stabilize grid freq, but do not return  $\delta$  to nominal 60Hz.

AGC { Load Freq. Control [Droop]  
Economic Dispatch.

$ACE = \Delta P_{tie} + B_f \Delta f$   
 $\uparrow$  Freq Bias Const maybe =  $\beta$  but not always.  
should be zero

Economic Dispatch

$C_i(P_{Gi}) = K_i + \beta P_{Gi} + \gamma P_{Gi}^2 \left[ \frac{\$/hr}{MW} \right]$

$IC_i(P_{Gi}) = \frac{dC_i(P_{Gi})}{dP_{Gi}} = \beta + 2\gamma P_{Gi} \left[ \frac{\$/hr}{MW} \right]$

Minimize  $G = \sum_{i=1}^n C_i(P_{Gi})$  such that  $\sum_{i=1}^n P_{Gi} = P_D + P_{losses}$

Econ. DISP Lagrange

$P_D = P_{G1} + P_{G2} = 500 \text{ MW}$   
 $C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2$   
 $C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2$

$\frac{dC_1(P_{G1})}{dP_{G1}} - \lambda = 20 + 0.02P_{G1} - \lambda = 0$

$\frac{dC_2(P_{G2})}{dP_{G2}} - \lambda = 15 + 0.06P_{G2} - \lambda = 0$

$500 - P_{G1} - P_{G2} = 0$

$\begin{bmatrix} 0.02 & 0 & -1 \\ 0 & 0.06 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -20 \\ -15 \\ -500 \end{bmatrix}$

$\begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} 312.5 \text{ MW} \\ 187.5 \text{ MW} \\ 26.2 \$/\text{MW} \end{bmatrix}$

Penalty factor less  $\rightarrow$  gen. (approx w/ small  $\lambda$ )  
 $L_i = \left( \frac{1}{1 - \frac{\partial P_e}{\partial P_{Gi}}} \right)$

Econ. Dispatch

- Does not turn on/off. [unit commitment]
- Ignores T system limit.

OPF - Optimal Power Flow takes into account Transmission eq 9.

- Gen Voltage Setpoints
- Real and Reactive Power
- Generator Limits
- Conservation of Energy.

Controls

- Gen MW outputs
- Transformer taps + Phase  $\theta$

Locational Marginal Price

- Sys. marginal cost ( $\lambda$ ) same all over, unless congestion
- w/ congestion price varies along network, LMP = nodal price.

Ancillary Services

- Frequency regulation
- Spinning Reserve
- Replacement Reserve
- Supplemental Reserve
- Voltage control

Grid Quality

- Stability - return to equilibrium
- Security - (N-1) sudden loss.
- Reliability - Actual interruption.
- Resiliency -
- IROL - Contingencies outside area.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

Energy conserved  $\Rightarrow P_1 = P_2$

$$I_2 V_2 = I_1 V_1$$

Transformer - 20 kVA  
 - 480/120V  
 - 60 Hz

Leakage Impedance  $Z_{eq2} = .0525 \angle 78.13^\circ \Omega$   
 @ winding 2

$$S_b = 20 \text{ kVA} \quad V_{b1} = 480 \text{ V} \quad V_{b2} = 120 \text{ V}$$

$$Z_{base2} = \frac{V_{b2}^2}{S_{base2}} = \frac{120^2}{20,000} = .72 \Omega$$

$$Z_{eq2pu} = \frac{Z_{eq2}}{Z_{base2}} = \frac{.0525 \angle 78.13}{.72} = .0729 \angle 78.13 \text{ pu}$$

$$Z_{eq1} = a^2 Z_{eq2} = \left(\frac{480}{120}\right)^2 (.0525 \angle 78.13) = .84 \angle 78.13 \Omega$$

$$Z_{base1} = \frac{V_{base1}^2}{S_{base1}} = \frac{480^2}{20,000} = 11.52 \Omega$$

$$Z_{eq1pu} = \frac{Z_{eq1}}{Z_{base1}} = \frac{.84 \angle 78.13}{11.52}$$

$$Z_{eq2pu} = .0729 \angle 78.13 \text{ pu}$$

Transformer Simplification

