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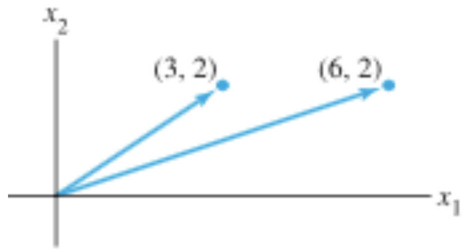
Linearly independent

- when the vector equation $x_1\mathbf{v_1} + x_2\mathbf{v_2} + \ldots + x_n\mathbf{v_n} = \mathbf{b}$ of an indexed set of vectors $\{\mathbf{v_1} \ldots\ldots \mathbf{v_p}\}$ has *only* the trivial solution (where **x** is the 0 vector)
- A set of ***two vectors*** {v1, v2} is linearly independent if and only if neither of the vectors is a multiple of the other

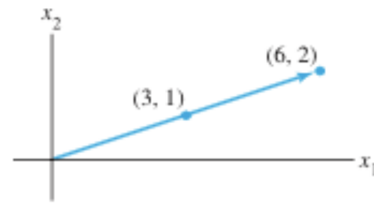


Linearly independent

- No free variables
- Given a set S = $\{\mathbf{v_1} \ldots \mathbf{v_p}\}$, no member in the set can be written as a **linear combination** of the rest
- **Invertible**

Linearly dependent

- when there exist weights $c_1, \ldots, c_p$, not *all* zero, such that $c_1\mathbf{v_1} + c_2\mathbf{v_2} + \ldots + c_p\mathbf{v_p} = \mathbf{0}$
  - By definition, **not unique and not invertible**
  - A set of two vectors $\{\mathbf{v_1}, \mathbf{v_2}\}$ is linearly dependent if at least one of the vectors is a multiple of another



Linearly dependent

  - The above equation is called a **linear dependence relation** when the weights are not all zero.
  - If a set contains more vectors (*columns*) than entries in each vector (*rows*), then the set is linearly dependent (**i.e. p > n**)
    - **This would mean that there is a free variable**
  - An indexed set S = $\{\mathbf{v_1} \ldots\ldots \mathbf{v_p}\}$ is linearly dependent if and only if *at least one* of the vectors in S is a linear combination of the others
    - Only one needs to be to satisfy this requirement. A vector in a linearly dependent set may fail to be a linear combination of the others as long as at least one of the vectors in the set is.
  - If a set S = $\{\mathbf{v_1} \ldots\ldots \mathbf{v_p}\}$ contains the zero vector, it is linearly dependent
  - If **u** and **v** are linearly *independent*, the set **{u, v, w}** will be linearly *dependent* if and only if **w** is in the plane spanned by, meaning it is a linear combination of, **u** and **v**