

MIDTERM 2 – STUDY GUIDE

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Note: Midterm 2 covers sections 3.4 – 4.9 (except sections 3.11, 4.6, 4.8, and pages 224 – 230 of section 3.7).

Note: 3.4.7 means 'problem 7 in section 3.4'

Formulas to remember:

Here's an important list of formulas/tricks to remember for related rates and optimization (but **CAREFUL**, this list is not exhaustive!!!) :

- Formula for the distance between two points (x, y) and (x_0, y_0) :

$$D = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

- Pythagorean theorem
- Definition of sin, cos and tan (SOHCAHTOA)
- Law of similar triangles
- Law of cosines: $BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos(\angle BAC)$
- Addition law for cosines: $\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)$
- Addition law for sines: $\sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B)$
- Formulas for areas and/or volumes:
 - Volume of a cone: $V = \frac{\pi}{3} r^2 h$
 - Volume of a cylinder: $V = \pi r^2 h$
 - Surface area of a cylinder (without top and bottom): $S = 2\pi r h$
 - Volume of a ball: $V = \frac{4}{3} \pi r^3$
 - Surface area of a sphere: $S = 4\pi r^2$
 - Arclength of a sector of angle θ and radius r : $L = r\theta$
 - Area of a sector of angle θ and radius r : $A = \frac{1}{2} r^2 \theta$

CHAPTER 3: DIFFERENTIATION RULES

- Differentiate functions using the chain rule (3.4.7, 3.4.17, 3.4.21, 3.4.45, 3.4.49)
- Find the equations of tangent lines using the chain rule (3.4.53, 3.4.59)
- Find values of derivatives of functions given a graph or some other info (3.4.61, 3.4.63, 3.4.66, 3.4.68, 3.4.71, 3.4.72, 3.4.74, 3.4.97; also see Prof. Steel's practice exam)
- Differentiate formulas using implicit differentiation (3.5.9, 3.5.11, 3.5.19, 3.5.77)
- Use implicit differentiation to find an equation of the tangent line to a given curve at a given point (3.5.25, 3.5.29, 3.5.32)
- Find derivatives of functions involving inverse trig functions (3.5.51, 3.5.54, 3.5.60)
- Prove the formulas for the derivatives of \sin^{-1} , \cos^{-1} and \tan^{-1}

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- Solve miscellaneous problems involving implicit differentiation (**3.5.44, 3.5.45, 3.5.46, 3.5.77**)
- Find derivatives of functions involving \ln (3.6.11, 3.6.20, 3.6.21)
- Use logarithmic differentiation to find the derivative of a function (3.6.41, 3.6.43, 3.6.44, **3.6.51, 3.6.52**)
- For section 3.7, know that the derivative is a rate of change. In particular, the derivative of position is velocity, of velocity is acceleration, of mass is density, of cost is marginal cost, etc. (3.7.1, 3.7.8, 3.7.28, 3.7.31, 87 on page 266)
- Solve the differential equation $y' = ky$ subject to various conditions (for example, solve $y' = 2y$ with $y(3) = 1$)
- Solve word problems involving exponential growth and decay or compounded interest (3.8.3, 3.8.9, 3.8.11, 3.8.19, 94 on page 267)
- **Solve related rates problems** (For example, try out 3.9.5, 3.9.13, 3.9.15, 3.9.25, 3.9.27, 3.9.33, 3.9.40, 3.9.45, 3.9.46, 99 and 101 on page 267)
- Find the linearization $L(x)$ of a function f at a (3.10.1, 3.10.3)
- Find the differential of a function (3.10.11, 3.10.13)
- Use linear approximations and/or differentials to estimate a given number; also be able to say if your answer is an underestimate or an overestimate (3.10.23, 3.10.25, 3.10.26, 3.10.28, 3.10.43)
- Use differentials to estimate maximum errors or relative errors (3.10.35, 3.10.39, 105 on page 267)

CHAPTER 4: APPLICATIONS OF DIFFERENTIATION

- Given a graph, find the absolute and local maximum and minimum values of a function (4.1.5, 4.1.6)
- Sketch the graph of a function with given properties involving max/min values (4.1.7, 4.1.10, 4.1.11, 4.1.14)
- Find the critical numbers of a function (4.1.29, 4.1.34, 4.1.39, 4.1.41, 4.1.43 also check out 4.1.70)
- **Find the absolute max/min of a function on a given closed interval** (4.1.47, 4.1.51, 4.1.55, 4.1.57, 4.1.61, 4.1.62)
- Use the IVT and Rolle's theorem to show that an equation has exactly one solution (4.2.17, 4.2.18)
- Use Rolle's theorem to show that an equation has at most one or two zeros (4.2.19, 4.2.20)
- **Solve problems using the MVT** (4.2.23, 4.2.25, 4.2.26, 4.2.27, 4.2.28, 4.2.29, 4.2.34, 4.2.35, 4.2.36, 4.3.77)
- Show that an identity holds by differentiating both sides of the identity and finding C (4.2.32, 4.2.33)
- Given a graph, say where f is increasing, decreasing, concave up/down (4.3.1)
- Find intervals of increase/decrease, local max/min, intervals of concavity and inflection points, and horizontal and vertical asymptotes of a given function (4.3.9, 4.3.12, 4.3.13, 4.3.33, 4.3.45, 4.3.49, 4.3.51)
- Sketch the graph of a function with given properties involving first and second derivatives (4.3.25, 4.3.27)

- Evaluate limits using l'Hopital's rule (any problem in section 4.4 works, try out 4.4.11, 4.4.13, 4.4.17, 4.4.21, 4.4.29, 4.4.31, 4.4.33, 4.4.39, 4.4.45, 4.4.49, 4.4.51). Always remember to check the indeterminate form first, and see if there is an easier way to solve the problem!
- Use l'Hopital's rule to evaluate indeterminate powers (4.4.58, 4.4.61, 4.4.65, 4.4.77)
- Also check out 4.4.85, 4.4.86, 4.4.88
- Use the DISAIC method to sketch the graph of a function (4.5.13, 4.5.41, 4.5.45, 4.5.49, 4.5.54)
- Find the equation of the slant asymptotes to a given curve (4.5.67, 4.5.71)
- Show that a function does not have a slant asymptote (for example, show that $\ln(x) - x$ does not have one at ∞)
- **Solve optimization problems** (any problem in section 4.7 would do, try out 4.7.13, 4.7.14, 4.7.21, 4.7.23, 4.7.24, 4.7.29, 4.7.32, 4.7.39, 4.7.48, 4.7.61, 4.7.67, 4.7.70, 4.7.76)
- Find the most general antiderivative of a given function (4.9.7, 4.9.17, 4.9.21)
- Find f given f' or f'' (4.9.25, 4.9.35, 4.9.41, 4.9.48)
- Find the position of a particle given its velocity or acceleration (4.9.63, 4.9.64, 4.9.75, 4.9.76, 4.9.77)