STUDY GUIDE FOR THE FINAL

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Note: The final will cover everything from section 1.1 to section 6.3 inclusive, **EXCEPT** sections 1.4, 2.1, 3.11, 4.6, and 4.8.

Formulas to remember:

Just as for midterm 2, here's an important list of formulas/tricks to remember for related rates and optimization (but **CAREFUL**, this list is not exhaustive!!!) :

- Formula for the distance between two points (x, y) and (x_0, y_0):
$$D = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$
- Pythagorean theorem
- Definition of $\sin$, $\cos$ and $\tan$ (SOHCAHTOA)
- Law of similar triangles
- Law of cosines: $BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos(\angle BAC)$
- Addition law for cosines: $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
- Addition law for sines: $\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
- Formulas for areas and/or volumes:
  - Volume of a cone: $V = \frac{\pi}{3}r^2h$
  - Volume of a cylinder: $V = \pi r^2 h$
  - Surface area of a cylinder (without top and bottom): $S = 2\pi rh$
  - Volume of a ball: $V = \frac{4}{3}\pi r^3$
  - Surface area of a sphere: $S = 4\pi r^2$
  - Arclength of a sector of angle θ and radius r: $L = r\theta$
  - Area of a sector of angle θ and radius r: $A = \frac{1}{2}r^2\theta$

Note: 1.5.4 means Problem 4 in section 1.5 !

(Turn page)

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Know how to:

1. Chapter 1: Functions and Models

Note: Don't spend *too* much time on this chapter, it's mostly meant to be a review of precalculus.

- Determine if a function is one-to-one, given a graph or a formula (1.6.5, 1.6.9)
- Find the domain and range of inverse functions (1.6.18)
- Find exact values of inverse trig functions, such as $\tan^{-1}(1)$ etc. (1.6.63, 1.6.65)
- Simplify expressions involving inverse trig functions, using the triangle method (1.6.70)

2. Chapter 2: Limits and Derivatives

- **Find limits of a function:**
  - Step 1: Just by plugging in (2.3.3)
  - Step 2: By noticing that it's of the form $\frac{1}{0^+} = \infty$ or $\frac{1}{0^-} = -\infty$ (2.2.29, 2.2.33)
  - Step 3: By factoring out the numerator and the denominator and simplifying (2.3.11, 2.3.15, 2.3.19, 2.3.24)
  - Step 4: Whenever there is a square root, by multiplying numerator and denominator by the conjugate form (2.3.21, 2.3.25, 2.3.29, 2.3.30)
  - Step 5: By using the squeeze theorem (2.3.39, 2.3.40)
  - Step 6: By calculating $\lim_{x\to a^-}$ and $\lim_{x\to a^+}$ and by noticing that they're equal or not (2.3.13, 2.3.41, 2.3.45, 2.3.49)
  - Step 7: By using l'Hopital's rule and/or the $\ln$-trick for things that are of the form 0^0, ∞^0 and 1^∞ (see Chapter 4)

  Note: If you need more practice, try the following set of problems: 2.3.32, 2.3.42, 2.3.59, 2.3.62
- Find limits using the $\epsilon - \delta$ notion of a limit (2.4.20, 2.4.25, 2.4.29, 2.4.30) (don't focus **too** much on this! The final covers other, more important topics as well)
- Evaluate limits using continuity (2.5.36, 2.5.38)
- Show that an equation has at least one solution, using the Intermediate Value Theorem (2.5.51, 2.5.53, 2.5.54)
- **Find limits at infinity of a function:**
  - Step 1: Just by plugging in (2.6.28, 2.6.38)
  - Step 2: By factoring out the highest power out of an expression (2.6.31)
  - Step 3: By factoring out the highest power of the numerator and the denominator (2.6.17, 2.6.19, 2.6.21, 2.6.34)
  - Step 4: By factoring out the highest power of x out of a square root (2.6.22, 2.6.23, 2.6.24)
  - Step 5: By using the conjugate form, making sure to do Step 4 first (2.6.25, 2.6.26, 2.6.27, 2.6.39(c))
  - Step 6: By using the squeeze theorem (2.6.37, 2.6.57, 2.6.61)

  - Step 7: By using l'Hopital's rule and/or the $\ln$-trick for things that are of the form 0^0, ∞^0 and 1^∞ (see Chapter 4)

  Note: If you need more practice, try the following set of problems: 2.6.29, 2.6.33, 2.6.44
- Find the equation of the tangent line to a function at a point (2.7.5, 2.7.7, 2.7.18)
- Find the derivative of a function at a given point or in general, using the definition of the derivative (2.7.27, 2.7.30, 2.7.53, 2.8.19, 2.8.23, 2.8.27, also try out $\cos$ and $\sin$)
- Recognize a limit as the derivative of a function (2.7.33, 2.7.37)
- Remember that differentiability implies continuity, but that there are functions which are continuous, but not differentiable

3. Chapter 3: Differentiation Rules

- Find the derivative of a function, using the power rule, the product/quotient rules or the chain rule (hopefully you're already comfortable with this by now! 3.3.9, 3.4.21, 3.4.45, 3.5.51, 3.5.54, 3.6.31)
- Find the derivative of a function using implicit differentiation (3.5.17, 3.5.19)
- Find the derivative of a function of the form $f(x)^{g(x)}$ using logarithmic differentiation (3.6.39, 3.6.43, 3.6.49, 3.6.51, 3.6.52)
- Find the equation to the tangent line to a given curve at a given point (3.4.53, 3.5.27, 3.5.29, **3.5.44, 3.5.45, 3.5.46**)
- Find limits involving $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ and $\lim_{x\to 0} \frac{\cos(x)-1}{x} = 0$ (3.3.39, 3.3.40, 3.3.46)
- Given the position function of a particle, find its velocity and acceleration (87 on page 266); that's the only application you need to know from section 3.7
- Know the solution to $y' = ky$, if in doubt, Ce^{kt} out! :), and solve problems involving exponential growth and decay (94 on page 267)
- **Solve related rates problems** (For example, try out try out 3.9.5, 3.9.13, 3.9.15, 3.9.25, 3.9.27, 3.9.33, 3.9.40, 3.9.45, 3.9.46, 99 and 101 on page 267)
- Find the linear approximation of a function at a given point (3.10.2, 3.10.4)
- Find the differential dy of a function (3.10.15)
- Use a linear approximation or differentials to estimate a given number (3.10.23, 3.10.25, 3.10.27)
- Use differentials to estimate the maximum and the relative error (3.10.35, 3.10.36, 3.10.43)

4. Chapter 4: Applications of Differentiation

- Find the absolute maximum and minimum values of a function on a given interval (4.1.41, 4.1.47, 4.1.51, 4.1.57, 4.1.63)
- Using Rolle's theorem (and the Intermediate Value Theorem), show that an equation has exactly one solution, or at most one solution, etc. (4.2.17, 4.2.18, 4.2.19, 4.2.22)

- Solve problems using the Mean Value Theorem (4.2.23, 4.2.24, 4.2.25, 4.2.27, 4.2.29, 4.2.34, 4.2.35, 4.2.36, 4.3.77, also show that if $f'(x) \neq 0$ for all x, then f is one-to-one)
- Show that an identity holds by differentiating it, and evaluating it at one point (4.2.32, 4.2.33)
- Evaluate limits using l'Hopital's rule (any problem in section 4.4 works, try out 4.4.11, 4.4.13, 4.4.17, 4.4.21, 4.4.29, 4.4.31, 4.4.33, 4.4.39, 4.4.45, 4.4.49, 4.4.51). Always remember to check the indeterminate form first, and see if there is an easier way to solve the problem!
- Use l'Hopital's rule to evaluate indeterminate powers $0^0, \infty^0, 1^\infty$ (4.4.57, 4.4.58, 4.4.61, 4.4.65)
- **Use the DISAIC method to sketch the graph of a function** (4.3.33, 4.3.43, 4.5.15, 4.5.25, 4.5.39, 4.5.49, 4.5.54, also try out $y = e^{\frac{1}{x}}$), don't worry about slant asymptotes, and remember that you can always check your answer with a calculator!
- **Solve opimization problems** (any problem in section 4.7 would do, try out 4.7.13, 4.7.14, 4.7.21, 4.7.23, 4.7.24, 4.7.29, 4.7.32, 4.7.39, 4.7.48, 4.7.61, 4.7.67, 4.7.70, 4.7.76)
- Find the position of a particle, given its velocity or its acceleration (4.9.59, 4.9.63)
- Don't worry too much about section 4.9, given that it's just a prerequisite to chapter 5.

5. Chapter 5: Integration

- **Given n** (say $n = 5$), Estimate the integral of a function, using right endpoints, left endpoints, or midpoints (5.1.2, 5.1.5, 5.2.3)
- Express the area under the graph of a given function as a limit (5.1.19, 5.1.20, 5.1.21)
- Determine a region whose area is equal to a given limit (5.1.22, 5.1.23)
- Express a limit as a definite integral on a given interval (5.2.17, 5.2.20)
- **Evaluate integrals directly, using the definition of an integral** (5.2.21, 5.2.22, 5.2.23, 5.2.24, 5.2.28)
- **Note:** On the exam, we will give you the formulas for $\sum_{i=1}^{n} i$ etc. so you don't have to worry about memorizing them.
- Evaluate integrals by interpreting them in terms of areas (5.2.35, 5.2.37, 5.2.39)
- Use the comparison property of integrals to prove an identity involving integrals (5.2.56, 5.2.57)
- Express a limit as a definite integral and evaluate that integral (5.2.71, 5.2.72, 5.3.69, 5.3.70)
- **Use the FTC Part I to find the derivative of a function** (5.3.7, 5.3.11, 5.3.15, 5.3.17, 5.3.55, 5.3.57, 5.3.59, 5.3.61)
- Use FTC Part I to write a formula for the antiderivative F of a given function f (say $f(x) = \sin\left(x^2\right)$) that satisfies a certain value, say $F(3) = 5$)
- As a consequence of the above problem, every continuous function has an antiderivative

- **Use the FTC Part II to evaluate integrals** (any problem between 5.3.19 and 5.3.44, as well as between 5.4.21 and 5.4.46, would do, try out 5.3.25, 5.3.31, 5.3.35, 5.3.39, 5.3.42, 5.3.43, 5.4.29, 5.4.37, 5.4.38, 5.4.45, 5.4.46 for example)
- Find indefinite integrals/antiderivatives (5.4.9, 5.4.12, 5.4.18)
- Interpret what definite integrals represent in real life (5.4.51, 5.4.54)
- Given a velocity or acceleration, find the displacement and the distance traveled by a particle during a given time interval (5.4.60, 5.4.61)
- Evaluate integrals by using the substitution rule (any problem between 5.5.7 and 5.5.48, as well as between 5.5.53 and 5.5.73 would do, try out 5.5.17, 5.5.21, 5.5.23, 5.5.33, 5.5.43, 5.5.45, 5.5.46, 5.5.59, 5.5.67 for example and definitely look at the integral on Prof. Steel's practice final)
- Remember that if f is odd, then $\int_{-a}^{a} f(x)dx = 0$ (5.5.66, 5.5.77)

6. Chapter 6: Applications of integration

- Find the area of the region enclosed by the given curves (6.1.8, 6.1.9, 6.1.11, 6.1.13, 6.1.17, 6.1.21, 6.1.26)
- Evaluate an integral involving absolute values (6.1.31)
- Find a number such that a given line divides a region into two regions with equal area (6.1.51, 6.1.52, 6.1.53)
- Find a volume using the disk or washer method (6.2.5, 6.2.6, 6.2.9, 6.2.11, 6.2.14, 6.2.17, 6.2.27, 6.2.28, 6.2.29, 6.2.30)
- Find volumes of more sophisticated solids using the disk or washer method (6.2.47 6.2.48, 6.2.49, 6.2.65)
- Find a volume using the **definition** of the volume, i.e. find $A(x)$ explicitly (6.2.51, 6.2.52, 6.2.54, 6.2.55, 6.2.56)
- Solve the related rates problem about volumes that Prof. Steel covered in lecture (the one involving water pouring into a bowl)
- Find the volume of a solid using cylindrical shells (6.3.3, 6.3.5, 6.3.13, 6.3.15, 6.3.19)
- Find the volume of more sophisticated solids using the shell method (6.3.45, 6.3.46, 6.3.47, 6.3.48)
- As a grand finale, evaluate the volume of a solid using any of the 3 methods taught in this chapter (6.3.37, 6.3.38, 6.3.39, 6.3.40, 6.3.41, 6.3.42)