

1-D, steady state sol to heat eqn. $q \neq 0$

	Plane Wall	Cylindrical Wall	Spherical Wall
Heat eq.	$\frac{d^2 T}{dx^2} = 0$	$\frac{1}{r} \frac{d}{dr} (r \frac{dT}{dr}) = 0$	$\frac{1}{r^2} \frac{d}{dr} (r^2 \frac{dT}{dr}) = 0$
Temp Dist.	$T_{s,1} - \Delta T \frac{x}{L}$	$T_{s,2} + \Delta T \frac{\ln(r_1/r)}{\ln(r_2/r)}$	$T_{s,1} - \Delta T \left[\frac{1 - (r_1/r)}{1 - (r_1/r_2)} \right]$
q''	$k \frac{\Delta T}{L}$	$\frac{k \Delta T}{r \ln(r_2/r)}$	$\frac{k \Delta T}{r^2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)}$
q	$kA \frac{\Delta T}{L}$	$\frac{2\pi L k \Delta T}{\ln(r_2/r)}$	$\frac{4\pi k \Delta T}{(r_1) - (1/r_2)}$
$R_{t,rad}$	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi L k}$	$\frac{4\pi k}{(r_1) - (1/r_2)}$

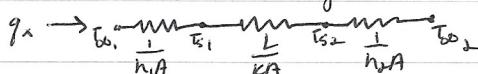
Boundary Conditions

- Constant $T_s \rightarrow T(0, t) = T_s$
- Constant $q'' \rightarrow$ Finite flux $\rightarrow q''|_{x=0} = -k \frac{\partial T}{\partial x}|_{x=0} = q''_s$
Adiabat/Insulated $\rightarrow \frac{\partial T}{\partial x}|_{x=0} = 0$
- Convective surface.
 $\rightarrow -k \frac{\partial T}{\partial x}|_{x=0} = h [T_{\infty} - T(0, t)]$

Thermal Resistance

- $R_{t,cond} = \frac{L}{kA}$
- $R_{t,conv} = \frac{T_s - T_{\infty}}{hA}$
- $R_{t,rad} = \frac{T_{surr} - T_{surv}}{q_{rad}}$
- Can write q as sum
i.e. $\left\{ q_x = \frac{T_{\infty} - T_{surr}}{\frac{1}{hA} + \frac{L}{kA} + \frac{1}{h_r A}} \right\}$ plane wall
w/ convection.

Draw resistance diagrams.



Temp. dist. and heat loss for first/last cross sec

case	Temp Condition	Temp. dist. θ/θ_b	Heat rate q_f
A	Convective heat transfer $h\theta(L) = -k d\theta/dx _{x=L}$	$\frac{\cosh m(L-x)}{\cosh mL} + \frac{(h/mk) \sinh m(L-x)}{(h/mk) \sinh mL}$	$M \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$
B	Adiabatic $d\theta/dx _{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M \tanh(mL)$
C	Prescribed T. $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b) \sinh mx + \sinh m(L-x)}{\sinh mL}$	$M \frac{(\cosh mL - \frac{\theta_L}{\theta_b})}{\sinh mL}$
D	∞ fin $\theta(L) = 0$	e^{-mx}	M

$$\theta = T - T_{\infty}$$

$$m^2 = hP/kA$$

$$\theta_b = \theta(0) = T_b - T_{\infty}$$

$$M = ThPKAC^2 \theta_b$$

Fin Effectiveness

$$\epsilon_f = q_f / (hA_{c,b} \theta_b) \quad \text{where } A_{c,b} = \text{fin cross sec. at base.}$$

$$\text{Resistance} \rightarrow R_{t,f} = \theta_b / \epsilon_f \quad \left\{ \epsilon_f = \frac{R_{t,b}}{R_{t,f}} \right\}$$

$$\text{R at base} \rightarrow R_{t,b} = 1/hA_{c,b}$$

$$\text{Max fin efficiency} \rightarrow \eta_f = \epsilon_f / \epsilon_{max} = q_f / (hA_f \theta_b)$$

Straight Fins

- Rectangular

$$A_f = 2wL_c$$

$$L_c = L + (t/2)L$$

$$A_p = tL$$

$$\eta_f = \frac{\tanh m L_c}{m L_c}$$

- Triangular

$$A_f = 2w(L^2 + (t/2)^2)^{1/2}$$

$$A_p = (t/2)L$$

$$\eta_f = \frac{1}{m t} \frac{I_0(2mL)}{I_0(2am)}$$

Shape Factor

$$\epsilon_f = S_k \Delta T_{i-2}$$

$$R_{t,rad} = \frac{1}{S_k}$$

Dimensionless Heat Cond. Rate

L_c for ∞ medium cases

$$L_c = \sqrt{\frac{As}{4\pi}}$$

$$\epsilon_f^* = \frac{q L_c}{KA_s(T_i - T_s)}$$