Finite Difference Equations

$$\frac{\partial^2 T}{\partial x^2}\bigg|_{m,n} = \frac{\partial T/\partial x|_{m+1,n} - \partial T/\partial x|_{m-1,n}}{\Delta x} \Big\} \text{ Evaluate similarly for } y.$$

$$= \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{\Delta x^2}$$

Eval Each node

$$q_{m+1,n \to m,n} = k\underbrace{(\Delta y \cdot 1)}_{\text{perpendicular area of contact.}} \frac{T_{m+1,n} - T_{m,n}}{\underbrace{\Delta x}_{\text{distance between nodes.}}}$$

Adjust for heat q'' and area, convection, etc.

Lumped Capacitance $B_i < 0.1$

- $\frac{\theta}{\theta_i} = \exp\left(-\left(\frac{hA_s}{\rho V c}\right)t\right) = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-B_i \cdot F_o)$
- $\tau_t = \left(\frac{1}{hA_s}\right)\rho V c = R_t C_t$
- $B_i = \frac{hL_c}{k}$ $F_o = \frac{\alpha t}{L_c^2}$ $\alpha = \frac{k}{\rho c}$
- General $\Rightarrow \frac{T - T_\infty - (b/a)}{T_i - T_\infty - (b/a)} = \exp(-at)$

$$a = \frac{hA_{s,c}}{\rho V c} \qquad b = \frac{q'' A_{s,h} + \dot{E}_{gen}}{\rho V c}$$

∞ Cylinder w/ Convection

- $B_i = \frac{hr_o}{k}$ $\theta^* = C_1 \exp(-\zeta_1^2 F_o) J_0(\zeta_1 r^*)$

$$\theta^* = \theta_o^* J_0(\zeta_1 r^*)$$

- Total Energy Transfer:

$$\frac{Q}{Q_o} = 1 - \frac{2\theta_o^*}{\zeta_1} J_1(\zeta_1)$$

Sphere w/ Convection

- $B_i = \frac{hr_o}{k}$ $\theta^* = \theta_o^* \frac{1}{\zeta_1 r^*} \sin(\zeta_1 r^*)$
- Total Energy Trasfer

$$\frac{Q}{Q_o} = 1 - \frac{3\theta_o^*}{\zeta_1^3}\left[\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)\right]$$

Thermal Penetration Depth

- $\delta(t) \triangleq 3.6\sqrt{\alpha t}$
- Large $\to$ semi-∞?

$$L > 3.6\sqrt{\alpha t_c}$$

Spatial Effects

$$\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} \qquad x^* = \frac{x}{L} \qquad t^* = \frac{\alpha t}{L^2} = F_o$$

$$\Rightarrow \theta^* = f(x^*, F_o, B_i)$$

Plane wall w/ Convection $F_o > 0.2$

- Approx Sol.

$$\theta^* = C_1 \exp(-\zeta_1^2 F_o)\cos(\zeta_1 x^*)$$

$$\theta^* = \theta_o^* \cos(\zeta_1 x^*)$$

where

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = C_1 \exp(-\zeta_1^2 F_o)$$

- Total Energy transfer

$$\frac{Q}{Q_o} = 1 - \frac{\sin \zeta_1}{\zeta_1}\theta_o^* \qquad \text{where } Q_o = \rho c V (T_i - T_\infty)$$

Semi-∞ solid

- Constant T_s: $T(0,t) = T_s$

$$\to \frac{T(x,t) - T_s}{T_i - T_s} = \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$\to q_s''(t) = \frac{k(T_s - T_\infty)}{\sqrt{\pi \alpha t}}$$

- Constant $q_s'' = q_o''$

$$\to T(x,t) = \frac{2q_o''(\alpha t/\pi)^{1/2}}{k}\exp\left(\frac{-x^2}{4\alpha t}\right) + T_i - \frac{q_o'' x}{k}\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

- Surface Convection

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0,t)]$$

$$\to \frac{T(x,t) - T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right)\right]\left[\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$