Induction

- Important B identities
  - Ampere's Law: $\oint B \bullet d\boldsymbol{l} = \mu_0 I$
  - For a wire, $d\boldsymbol{l} = 2\pi r$
- Causes – Change in I → E
- Faraday's Law
  - $\oint B \bullet d\boldsymbol{l} = -d/dt \int B \bullet dA$
  - $\mathcal{E} = d\phi_B / dt$
  - $\mathcal{E} = v_0 \boldsymbol{l} B = IR$
  - $I = v_0 \boldsymbol{l} B / R$
  - $P_{loop} = B^2 \boldsymbol{l}^2 v^2 / R$
- $\mathcal{E}$ only exist if $\Delta\phi_B$
- Lentz's Law
  - Currents (Inducted) counter a B field currently being transmitted to ensure cooperation of 1st law
- Equations
  - Angular Speed = $\omega = 2\pi f$
  - $v_{metal} = RMg / (\boldsymbol{l}B)^2$
- Self-Inductance
  - $L = \phi_B / I$
  - $\mathcal{E}_L = -L\,(di/dt)$
  - Solenoid
    - $\mu_0 n^2 A\boldsymbol{l}\,(di/dt) = \Delta V$
    - $L = \mu_0 n^2 A\boldsymbol{l}$
    - $B = \mu_0 n \boldsymbol{l}$
    - Energy Density: $u_B = B^2 / 2\mu_0$
    - $U = u_B A\boldsymbol{l}$
- Circuits
  - Current at Initial state = open circuit
  - Current at Final state = short circuit
  - Inductor current: $\mathcal{E}_L = -\mathcal{E}_0 e^{-Rt/L}$
    - $I = \mathcal{E}_{L+}\mathcal{E}_L / R = (\mathcal{E}_0/R)(1 - e^{-Rt/L})$
    - Loop Rule: $0 = \mathcal{E}_0 - IR - L\,(di/dt)$
  - Energy and Power
    - $U = ½ LI^2$
    - $P = LI\,(di/dt) = I^2R$
- Magnetic Force
  - $F = qv \times B$
  - $F = I\boldsymbol{l} \times B$
- Magnetic and Electric fields
- $\Delta\vec{B} = \Delta\vec{E}$
- $\oint E \bullet dr = -d/dt \int B \bullet dA$

Maxwell's Equation

- The 4 Equations
  - [Gauss E] $\oint E \bullet dA = q / \epsilon_0$
  - [Gauss B] $\oint B \bullet dA = 0$
  - [Amperé] $\oint B \bullet dr = \mu_0 I + \mu_0 \epsilon_0 (d\phi_E / dt)$
  - [Faraday] $\oint E \bullet dr = -(d\phi_B / dt)$
- When in vacuum, $q / \epsilon_0 = 0$ and $\mu_0 I = 0$
- $\vec{B}$ and $\vec{E}$
  - [Faraday] $\delta E / \delta x = -(\delta B / \delta t)$
  - [Amperé] $\delta B / \delta x = -\mu_0 \epsilon_0 (\delta E / \delta t)$

Properties of Light

- $\vec{B}$ and $\vec{E}$
  - $kE_p = \omega B_p$
  - $kB_p = \mu_0 \epsilon_0 \omega E_p$
  - $E = cB$
- Equations
  - Wave
    - $E(x,t) = E_p \sin(kx - \omega t)\ (\hat{j})$
    - $B(x,t) = B_p \sin(kx - \omega t)\ (\hat{k})$
  - Angular Velocity
    - $\omega = 2\pi / T$
    - $\omega = 2\pi f$
  - Period
    - $T = 2\pi / \omega$
  - Propagation Velocity
    - Wave speed $c = (\omega / k) = (1 / \sqrt{\mu_0 \epsilon_0}) = 3 \times 10^8$
    - $c = \lambda f$, $k = 2\pi / \lambda$
  - Average Energy
    - $u_{EM} = ½E^2\epsilon_0 + ½B^2\mu_0 = E^2\epsilon_0 = (B^2 / \mu_0)$
    - $\langle U_{EM} \rangle = ½E^2\epsilon_0$
    - $u_E = ½E^2\epsilon_0$
    - $u_B = B^2 / 2\mu_0$
- Poynting Vector
  - Average Poynting Vector (Light Intensity)
    - $\langle S \rangle = (E_pB_p) / 2\mu_0 = (U_{EM})c$
  - Expanding in Spheres
    - $S = P / 4\pi r^2$
  - Radiation Pressure
    - [Absorb] $P_{rad} = \langle S \rangle / c$
    - [Reflect] $P_{rad} = 2\langle S \rangle / c$
    - $F = -eE = -ecB$
- Polarization
  - Relationship to $\vec{E}$
  - Intensity relationship
    - [Malus] Intensity: $S = S_0 \cos^2\Theta$

Reflection and Refraction

- Law of Reflection
  - $\Theta = \Theta'$
- Speed of light in different mediums
  - $n = c/v$
- Snell's Law
  - $n_1 \sin\Theta_1 = n_2 \sin\Theta_2$
- Brewster (Polarizing) Angle
  - Perpendicular "reflected" wave to the refracted one
  - $\tan\Theta_p = n_2 / n_1$
- Critical Angle
  - The angle at which refraction → reflection
  - $\sin\Theta_c = n_2 / n_1$
- Change in wavelength through a prism
  - Different λ have different n values in a prism

Lens and Mirror

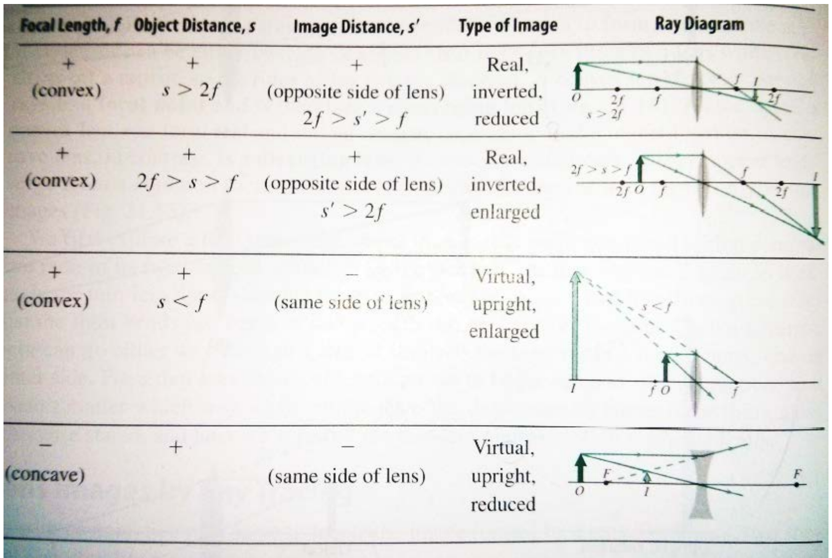
- Virtual vs Real Image:
  - Virtual is inferred by the brain, while real is projected light
- Mirrors vs Lenses
  - Convex : Diverging Lens :: Concave : Converging Lens, in terms of function and ray tracing\
  - Focal point: convergence of light, ½R = f
  - Spherical Abberation – minimized by making mirror a tiny fraction of a sphere (spherical vs parabolic)
- Ray Tracing
  - See figures
- s, s', and f relationship
  - $1/s + 1/s' = 1/f$
- Magnification
  - $M = h'/h = -(s'/s)$
- Lensmaker Equation (Thick Lenses)
  - $1/f = ((n_{lens}/n_{medium}) - 1)(1/R_1 - 1/R_2)$
- Refraction in a lens
  - $n_1/s + n_2/s' = (n_2 - n_1) / R$
  - n_1 = medium of object, n_2 = medium of other boundary, R = radius of lens/cylinder/sphere
- Optical instruments
  - Eyes
    - Divergent lenses – nearsightedness
    - Convergent lenses – farsightedness
    - Diopters = 1/f, [f] = meters
    - Angular Magnification (ratio of magnification due to correction)
      - m = 25(centimeters)/f
    - Compound Microscope
      - $M_0 m_e = -(L/f_0)(25\text{centimeters}/f_e)$
    - Telescope
      - $m = f_0 / f_e$

Constructive and Destructive Interference

- Interference derives from wave incoherence
- Paraxial approximation
  - When $\lambda \ll d$, $\text{trig}\Theta = \Theta$
- Wave Mutation Equation
  - If $\Phi = \pi$, cos = 0, Destructive interference; If $\Phi = 2\pi$ or 0, Constructive interference
  - $E_T = 2E\sin(kx - \omega t + \Phi/2)\cos(-(\Phi/2)$
  - $B_T = 2B\sin(kx - \omega t + \Phi/2)\cos(-(\Phi/2)$
- Double Slit Interference
  - Bright fringes: $d\sin\Theta = m\lambda$, (m = 0 (center),1,2,3…)
  - Dark fringes: $d\sin\Theta = (m + ½)\lambda$, (m = 0,1,2…)
  - $y_{bright} = (mL\lambda)/d$; $y_{dark} = ((m + ½)L\lambda)/d$
- Intensity Equation (double slit)
  - $\langle S \rangle = (4E_0^2/2\mu_0 c)\cos^2(d\pi y/\lambda L)$
- Multiple slit interference
  - N-1 minima between each pair of primary maxima
  - $d\sin\Theta = (m/N)\lambda$
  - Spectrometer
    - $d = 1/N$; N = number of slits
    - $d\sin\Theta = m\lambda$ (bright, separation)
- X ray Diffraction
  - $2d\sin\Theta = m\lambda$
- Resolving power
  - $\lambda/\Delta\lambda = mN$

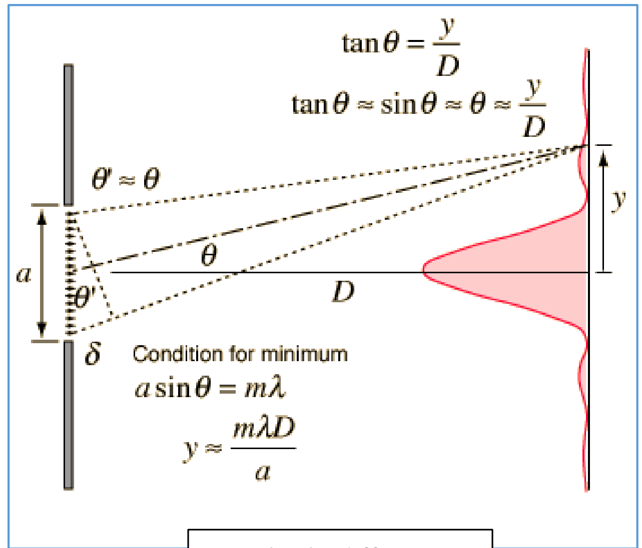
Diffraction

- Thin Film Optics
  - Speed of light in a medium: $v = c/n$
  - Frequency is constant
  - Phase shift of π when reflected off material with $n_{barrier}$ > current medium
  - No phase shift when $n_{barrier}$ < current medium
  - $2dn = (m + ½)\lambda$
- Hugyen's Principle
  - Circles – all points on a waveform act as point sources
  - Diffraction only truly happens when slit size is comparable to wavelength
  - $a\sin\Theta = m\lambda$ [destructive int, single-slit diffraction, (m = 1,2,3)]
- Intensity Equation (single slit)
  - $\langle S \rangle = (E_0^2/2\mu_0 c)\,(\sin(\Phi/2)/(\Phi/2))^2$; $\Phi = 2\pi a(\sin\Theta)/\lambda$
- Resolution
  - $1.22\,(\lambda / d) = \sin(\Theta_d)$

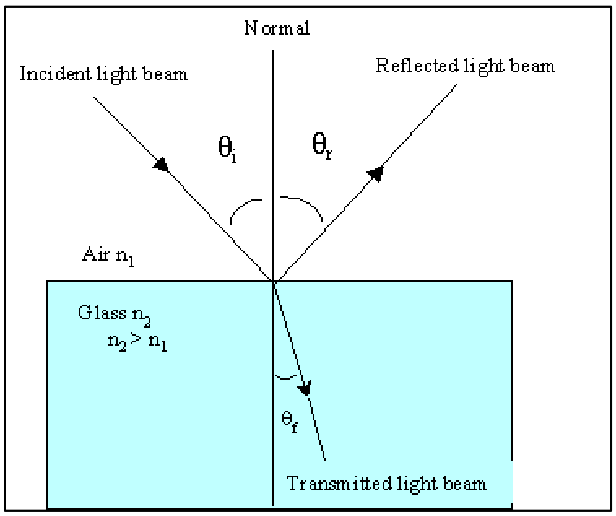


Focal Length, f	Object Distance, s	Image Distance, s'	Type of Image	Ray Diagram
+ (convex)	+ $s > 2f$	+ (opposite side of lens) $2f > s' > f$	Real, inverted, reduced	
+ (convex)	+ $2f > s > f$	+ (opposite side of lens) $s' > 2f$	Real, inverted, enlarged	
+ (convex)	+ $s < f$	− (same side of lens)	Virtual, upright, enlarged	
− (concave)	+	− (same side of lens)	Virtual, upright, reduced	

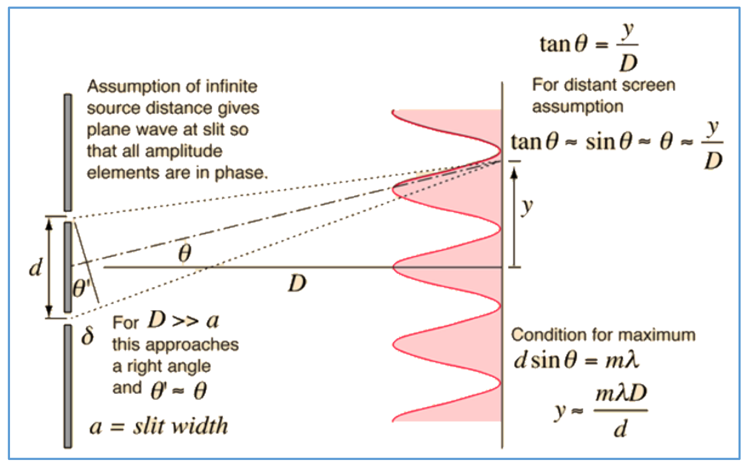
Lens Ray Tracing



Single slit diffraction



Refraction/Reflection



Double Slit Diffraction

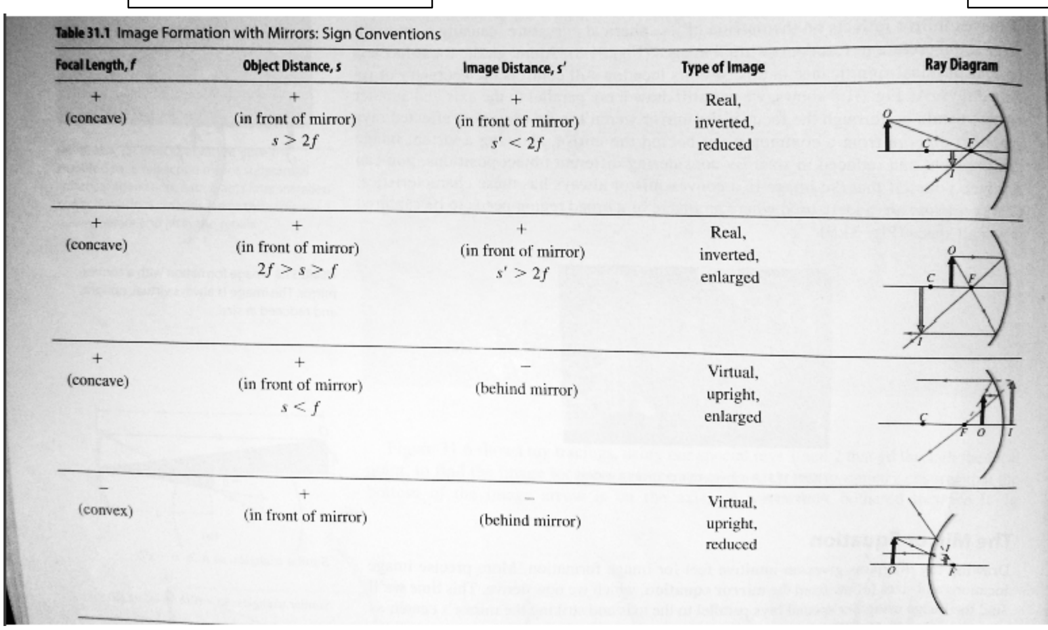


Table 31.1 Image Formation with Mirrors: Sign Conventions

Focal Length, f	Object Distance, s	Image Distance, s'	Type of Image	Ray Diagram
+ (concave)	+ (in front of mirror) $s > 2f$	+ (in front of mirror) $s' < 2f$	Real, inverted, reduced	
+ (concave)	+ (in front of mirror) $2f > s > f$	+ (in front of mirror) $s' > 2f$	Real, inverted, enlarged	
+ (concave)	+ (in front of mirror) $s < f$	− (behind mirror)	Virtual, upright, enlarged	
− (convex)	+ (in front of mirror)	− (behind mirror)	Virtual, upright, reduced	

Mirror Ray Tracing