Physics 7B Midterm 1 Review

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1 Zeroth Law of Thermodynamics

If two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.

2 Linear and Volume Expansion

2.1 Linear Expansion

$$L(T) = l_0 + l_0 \Delta T = l_0 (1 + \Delta T) \tag{1}$$

2.2 Volume Expansion

$$V(T) = L(T)W(T)H(T) = v_0(1 + 3\Delta T) \tag{2}$$

3 Gas Laws and Kinetic Theory

3.1 Energy

- Internal Energy = Total Energy

- Temperature = Average Energy

- Heat (Q) = Transfer of Energy

3.1.1 Kinetic Energy

For 1 particle:

$$E_{kin} = \frac{3}{2} K_B T \tag{3}$$

For N particles:

$$E_{kin} = \frac{3}{2} N K_B T = \frac{3}{2} nRT = \frac{3}{2} PV = \text{Internal Energy (for a gas)} \tag{4}$$

3.1.2 Velocity

$$V_{rms} = \sqrt{\frac{3k_B T}{m}} \tag{5}$$

3.2 Equipartition Theory

Gas uses its internal energy to move an equal amount in each degree of freedom. Therefore, if d is the number of degrees of freedom (3 for a monatomic ideal gas, 5 for a diatomic ideal gas):

$$\Delta U = \frac{d}{2} P_1 \Delta V = \frac{d}{2}(P_f V_f - P_i V_i) \qquad (6)$$

We can change this into the differential by replacing Δ with 'd'.

4 State Variables

A state variable is one that is path independent. In order for an integral to 'work', the relevant equation has to be path independent.

- ΔE_{int} is a state variable because it is at the same place at the beginning of each cycle.

- ΔS is a state variable.

5 First Law of Thermodynamics

$$\Delta U = \Delta Q - \Delta W \qquad (7)$$

Definitions:

- Q added: +

- Q removed/lost: -

- Work done **on** system (e.g. compressing gas): -

- Work done **by** system: +

5.1 Work

$$\Delta W = F \Delta X = PA \Delta X = P \Delta V \qquad (8)$$

- Work is only possible in the process of a *gradient* (Δ T, ΔP, etc)

- In the process of producing work, the gradient **diminishes** without external energy.

6 Heat Transfer

6.1 Conduction

6.1.1 Rate of heat transfer (Proportional to $\frac{A}{L}$)

$$\frac{\Delta Q}{\Delta T} = kA \frac{T_1 - T_2}{l} \qquad (9)$$

6.1.2   Equations

Differential Form (non-uniform cross-sectional Area):

$$\frac{dQ}{dT} = kA\frac{dT}{l} \tag{10}$$

6.1.3   Important Notes

A rod made of two materials has a uniform heat conductivity!

$$\frac{dQ}{dT}_{\text{Material 1}} = \frac{dQ}{dT}_{\text{Material 1}} \tag{11}$$

- **To calculate the temperature of the midpoint of a composite rod, the above equation cancels out to:**

$$\frac{k_1 \Delta T_1}{l_1} = \frac{k_2 \Delta T_2}{l_2} \tag{12}$$

  Where (assuming heat flow from left to right) ΔT is:

  $(T_h - T_1)$ on the left side and $(T_2 - T_L)$ on the right side.

- **To calculate the rate of heat flow of the composite rod:**

  Solve using either k_1 or k_2 with the temperature, T_m we found above

6.2   Radiation (probably not important)

$$\frac{\Delta Q}{\Delta T} = E\sigma AT^4 \sigma = 5.67 \times 10^{-8} \frac{W}{m^2 k^4} \tag{13}$$

7   Engines

For any engine, the larger the temperature gradient, the more work we can get out of it. In order for the engine to be a reversible process, we want to keep T_H and T_L the same to keep ΔT as large as possible throughout the process. ΔT can only be reduced, however, because it's a spontaneous process. This is why we want isothermal processes.

It is very important to keep in mind the signs of the heats exchanged in an engine. When the Carnot engine absorbs heat from a reservoir, the heat will be a positive quantity since the heat is being added to the engine, before it does any work. Similarly, when the Carnot engine rejects heat to a reservoir, the heat will be a negative quantity since the heat is lost from the engine. The work done by the engine, by the first law of thermodynamics, is therefore the sum of all heat changes in the engine.

7.1   Efficiency

All reversible engines operating between the same two constant temperatures T_H and T_L have the same efficiency. Any irreversible engine operating between the same two fixed temperatures will have an efticiency less than this.

$$e = 1 - \frac{T_L}{T_H} = 1 - \frac{Q_L}{Q_H} \tag{14}$$

7.2 For a Carnot (Reversible) Engine

Net entropy change of the engine during the process is zero. A Carnot engine operates reversibly, i.e. no entropy is produced in the process. Hence the entropy rise due to absorption of heat from hot reservoir (at temperature T_h) equals entropy loss due to heat rejection:

$$0 = \Delta S = \frac{Q_{in}}{T_h} - \frac{Q_{out}}{T_l} \tag{15}$$

Which can give us the ratio of:

$$\frac{Q_{in}}{Q_{out}} - \frac{T_h}{T_l} \tag{16}$$

8 2nd Law of Thermodynamics

8.1 Entropy

Entropy is the measure of the ability to do work.

$$\Delta S = \frac{\Delta Q}{\Delta T} \tag{17}$$

8.1.1 Integral Form

In general,

$$\Delta S => \int_{T_0}^{Tf} \frac{dQ}{T} = \int_{T_0}^{Tf} \frac{mc\Delta T}{T} = mcln(\frac{T_f}{T_0}) \tag{18}$$

$$\Delta S_{universe} = \Delta S_{gas} + \Delta S_{environment} \tag{19}$$

However, whether it is greater than or equal to 0 depends on whether the process is **reversible** or **irreversible**.

8.1.2 Reversible Processes

A process is reversible if Δ S = 0, or the ability to do work stays the same. **This is because, for irreversible processes, the work is produced by external heat added to the system.**
More specifically:

$$\Delta S = \int_{T_0}^{Tf} (\frac{dQ}{T})_{rev} = 0 \tag{20}$$

THIS ONLY HOLDS FOR REVERSIBLE PROCESSES!

8.1.3 Irreversible Processes

As stated in the main entropy section, entropy, ΔS, is the measure of the ability to do work. You can't use ΔS to calculate entropy of an irreversible process, because, for an irreversible process, the ability to do work is reduced. In **any** system, ΔS is a state variable, is path independent, and is not going to change. However, the integral for an irreversible system is path-dependent and is not equal to ΔS. **This is because, for irreversible processes, the work is produced by the gradient of the system.**

$$\Delta S > \int_{T_0}^{Tf} (\frac{dQ}{T})_{irrev} > 0 \tag{21}$$

8.1.4 Entropy of the Ideal Gas

The change of entropy between any two states of an ideal gas is given by the following formula, derived by considering an isobaric and isochoric expansion:

$$\Delta S = \frac{d}{2} N k_b ln(\frac{T_2}{T_1}) + N k_b ln(\frac{V_2}{V_1}) \tag{22}$$

In integral form, this is

$$\Delta S = \int_{T_0}^{T_f} c_p \frac{dT}{T} - Rln\frac{P_f}{P_0} \tag{23}$$

The first part is from an isobaric and the second part is from an isothermal.

9 Problem Solving Approaches

9.1 Heat Problem

9.1.1 Finding Amount of Heat Needed to Do Something

- When heat is added to an ideal monatomic gas at constant pressure:

$$Q = nC_p(T_f - Ti) = n\frac{5}{2}R(T_f - Ti) \tag{24}$$

- To solve for unknown n in an isobaric expansion with a weight
  (**Remember that $\mathbf{P} = \frac{F}{A}$!**):

$$n = \frac{V_iP_i}{RT_i} = \frac{mgV_i}{ART_i} \tag{25}$$

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9.2 Carnot Engine

9.2.1 General Hints

- Remember that the bottom reservoir is $Q_L and the top is Q_H, so think of how to find those Q susing heat of fusion/$

9.2.2 Find efficiency of a cycle

To find the efficiency $\frac{\Delta W}{Q_{in(H)}}$ of a cycle, break the heat engine (ABCD) into sections:

1. **Isothermal AB:** No change in temperature in an isothermal process, so $\Delta E_{int} = 0$, meaning:

$$Q_{AB} = W_{AB} = nRTln(\frac{V_2}{V_1}) = P_2V_2ln(\frac{V_2}{V_1}) \tag{26}$$

2. **Isovolumetric BC:**

   Because $\Delta V = 0, W = P\Delta V = 0$. Therefore:

$$\Delta E_{int} = Q = nC_v(T_2 - T_1) = \frac{d}{2}nR(T_2 - T_1) \tag{27}$$

3. **Adiabatic BC:** Very similar to isovolumetric except, in this case Q = 0, so:

$$\Delta E_{int} = W = nC_v(T_2 - T_1) = \frac{d}{2}nR(T_2 - T_1) \tag{28}$$

- For an engine to provide work if the area enclosed by the loop in a clockwise manner is positive: $T_1 > T_2$ for adiabatic portions and $V_2 > V_1$ for isothermal portions.

10 Hints

- Remember, there is no **horizontal** displacement when a partition is removed vertically, which means no work!

- You can always calculate Q for a constant volume monatomic gas process with

$$Q = nC_v(T_b - T_a) = \frac{3}{2}nR(T_b - T_a) \tag{29}$$

- One can relate the efficiency equations

$$e = 1 - \frac{T_L}{T_H} \text{ or } 1 - \frac{Q_L}{Q_H} \tag{30}$$

  with the definition of efficiency

$$e = \frac{W}{Q_0} = \frac{Q_0 - Q_1}{Q_0} \tag{31}$$

  to find

- Think of implicit constraints: if you're told the gas is in a rigid container, that means things like $C_v apply$!

- Remember, most things are in a **system**. This means that what applies to one applies to all. (Think of how logic forced us towards only thinking about the 'subsystem' of the disks when the change of internal energy really applied to more than that.

- Two items in contact have **EQUAL PRESSURE**!

- You can change V into A dx when calculating work if the cross-sectional area is constant and the change in x is kown (like in expansions).

- Something with a piston is usually isobaric because the weight of the piston provides a constant pressrue

- n is most likely constant, so if you know 2 out of the 3 $\frac{PV}{T}$, you can find the third.

- The T in the integral of entropy can be replaced by T in terms of volume, pressure, etc.

- dQ = 0 does not necessarily imply that change in entropy = 0. For a reversible process, change in entropy is zero. Entropy is actually a measure of how irreversible a process is.

- $C_p = \frac{5}{2}$R – don't forget the R

- Q is positive when 1

- To find heat used find W first and then find Q_L at low temperature and use the equation

$$W = Q_H - Q_L \tag{32}$$

- To find entropy change in any process, all you need to do is to chalk out an equivalent REVERSIBLE path for the process, and then calculate the heat change $\frac{dQ}{T}$ for that path. If integral of that $\int \frac{dQ}{T}$ is 0, then and only then, the process is isentropic.

- For Carnot Engine, heat exhausted at low temperature is $Q_L = -Q_{BC} - Q_{CD}$ and heat added is $Q_L = Q_{DA} + Q_{AB}$

- Always ask yourself: what temperature is this process occuring at. This will give insight as to whether it's a part of Q_H or Q_L because of

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